

# Molecular QCA and the limits of binary switching logic

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Marya Lieberman, Thomas Fehlner, Alex Kandel

Supported by NSF, State of Indiana



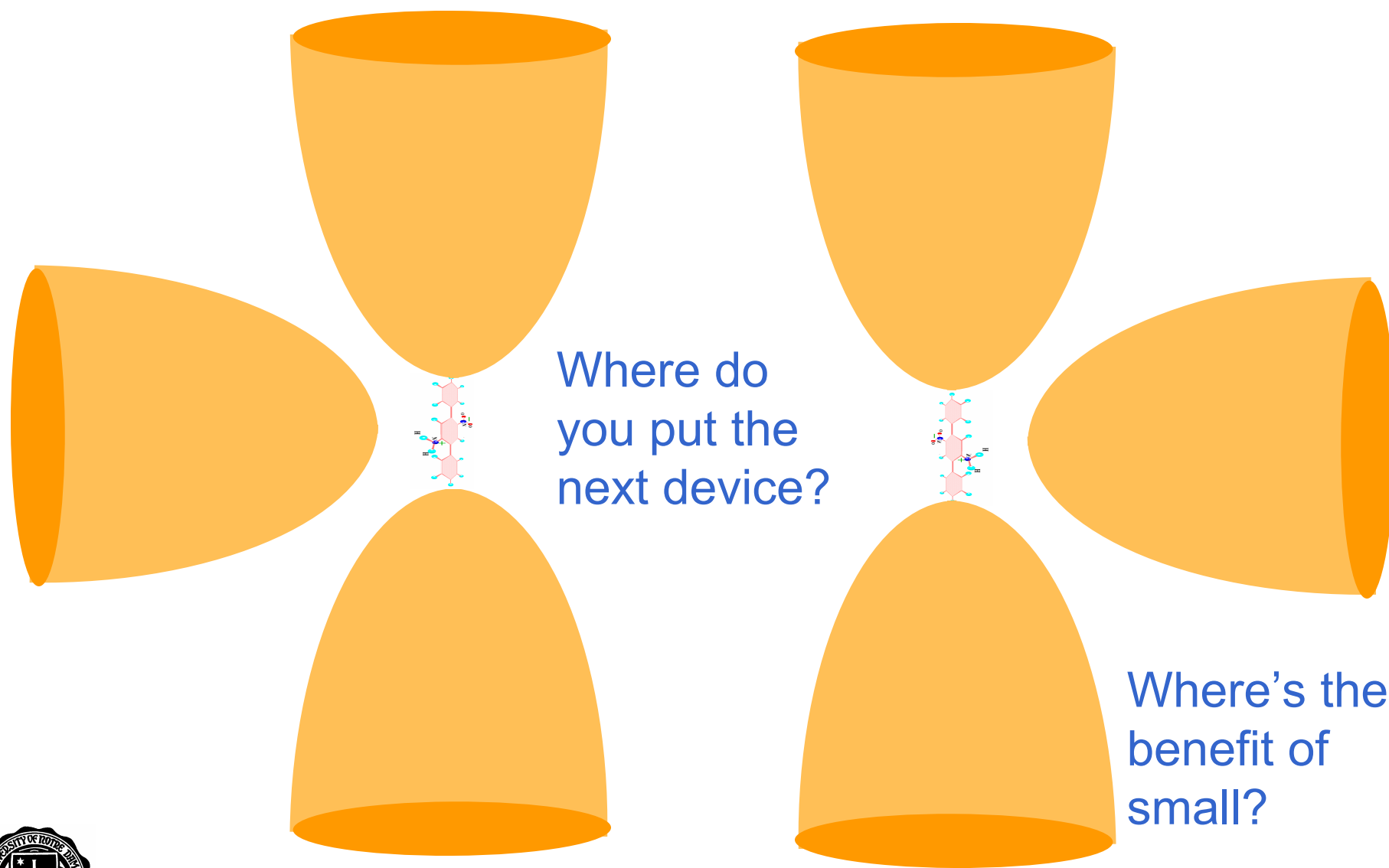
# The Dream of Molecular Transistors



Why don't we keep on shrinking transistors until they are each a single molecule?



# Molecular Transistors

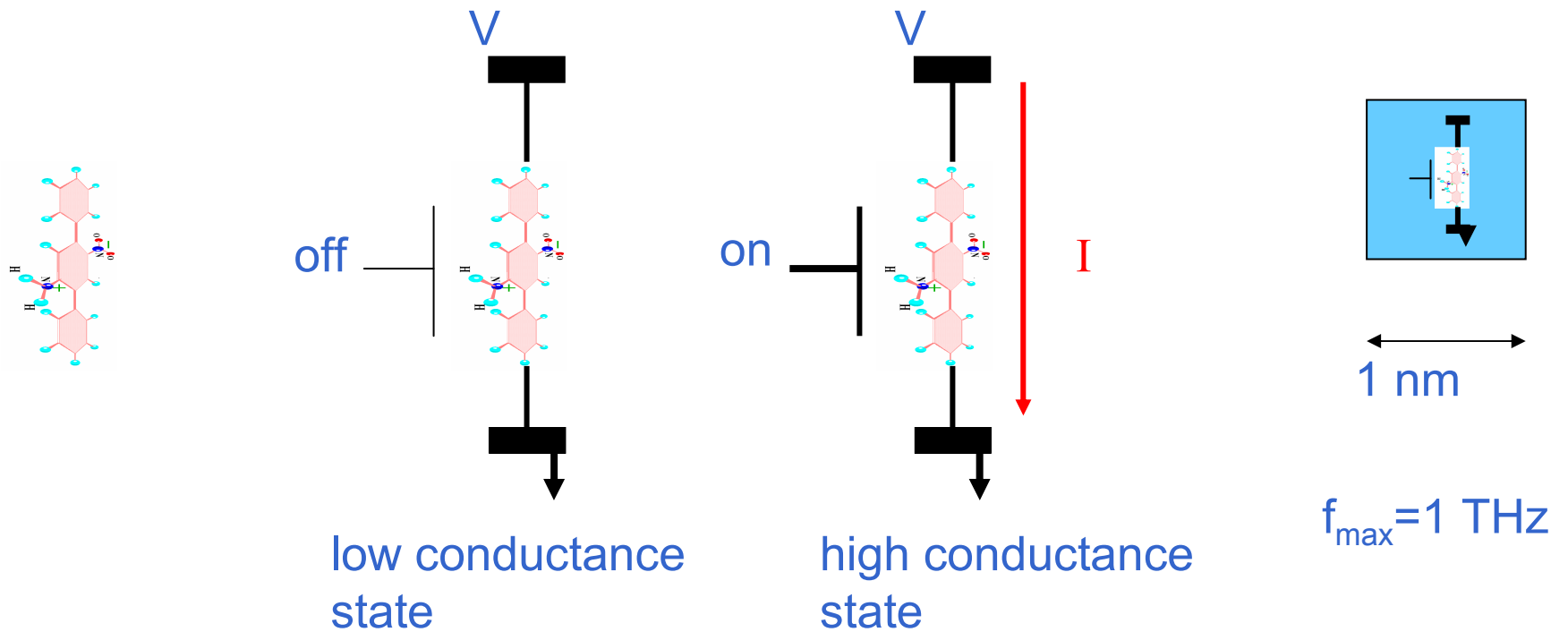


Where do  
you put the  
next device?

Where's the  
benefit of  
small?



# Dream molecular transistors

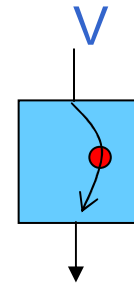


Molecular densities:  $1 \text{ nm} \times 1 \text{ nm} \rightarrow 10^{14} / \text{cm}^2$



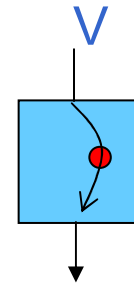
# Transistors at molecular densities

Suppose in each clock cycle a *single* electron moves from power supply (1V) to ground.



# Transistors at molecular densities

Suppose in each clock cycle a *single* electron moves from power supply (1V) to ground.



## Power dissipation (Watts/cm<sup>2</sup>)

Frequency (Hz)	10 <sup>14</sup> devices/cm <sup>2</sup>	10 <sup>13</sup> devices/cm <sup>2</sup>	10 <sup>12</sup> devices/cm <sup>2</sup>	10 <sup>11</sup> devices/cm <sup>2</sup>
10 <sup>12</sup>	16,000,000	1,600,000	160,000	16,000
10 <sup>11</sup>	1,600,000	160,000	16,000	1,600
10 <sup>10</sup>	160,000	16,000	1,600	160
10 <sup>9</sup>	16,000	1,600	160	16
10 <sup>8</sup>	1,600	160	16	1.6
10 <sup>7</sup>	160	16	1.6	0.16
10 <sup>6</sup>	16	1.6	0.16	0.016

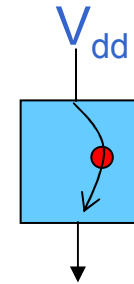
ITRS roadmap:

7nm gate length, 10<sup>9</sup> logic transistors/cm<sup>2</sup> @ 3x10<sup>10</sup> Hz for 2016



# Transistors at molecular densities

Suppose in each clock cycle a *single* electron moves from power supply (1V) to ground.



## Power dissipation (Watts/cm<sup>2</sup>)

Frequency (Hz)	$10^{14}$ devices/cm <sup>2</sup>	$10^{13}$ devices/cm <sup>2</sup>	$10^{12}$ devices/cm <sup>2</sup>	$10^{11}$ devices/cm <sup>2</sup>
$10^{12}$	16,000,000	1,600,000	160,000	16,000
$10^{11}$	1,600,000	160,000	16,000	1,600
$10^{10}$	160,000	16,000	1,600	160
$10^9$	16,000	1600	160	16
$10^8$	1600	160	16	1.6
$10^7$	160	16	1.6	0.16
$10^6$	16	1.6	0.16	0.016

ITRS roadmap:

7nm gate length,  $10^9$  logic transistors/cm<sup>2</sup> @  $3 \times 10^{10}$  Hz for 2016



# The Dream of Molecular Transistors





# Molecular electronics requirements

- 1) Low power dissipation
- 2) Real power gain
- 3) Robustness to disorder

Benefit: functional densities at molecular scale



# Outline

- Introduction
- QCA paradigm
- Implementations
  - Metal-dot QCA
  - Molecular QCA
- Energy flow in QCA
  - Power gain
  - Power dissipation and erasure
  - Bennett clocking

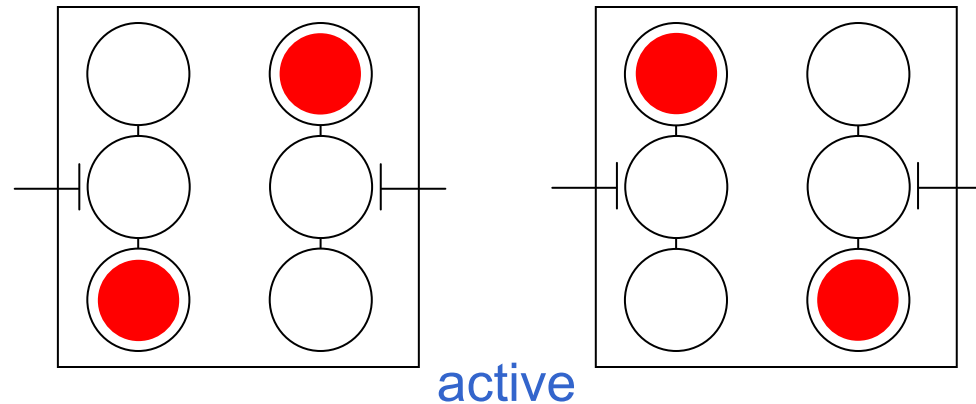


# Quantum-dot cellular automata

Represent binary information by charge configuration of cell.

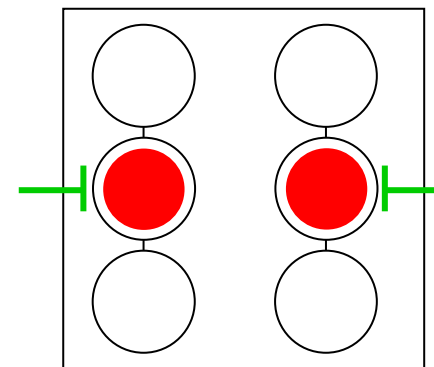
## QCA cell

- Dots localize charge
- Two mobile charges
- Tunneling between dots
- Clock signal varies relative energies of “active” and “null” dots



“1”

“0”

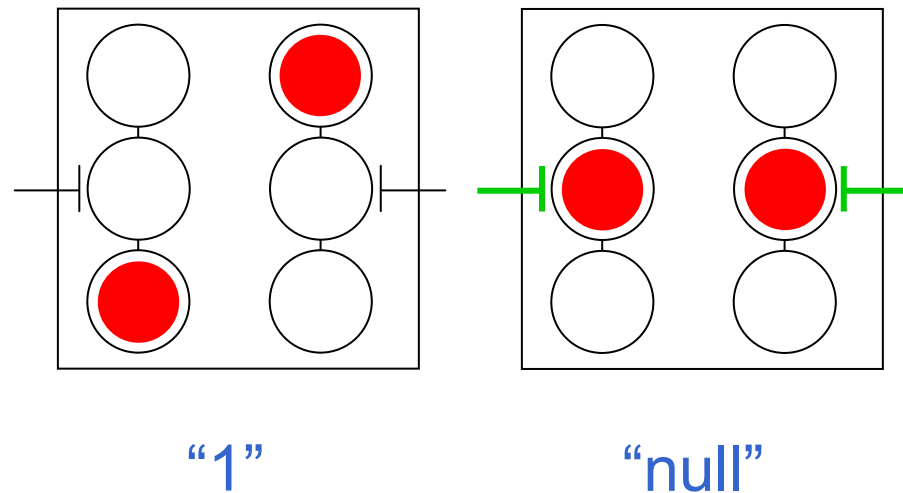


Clock need not separately contact each cell.



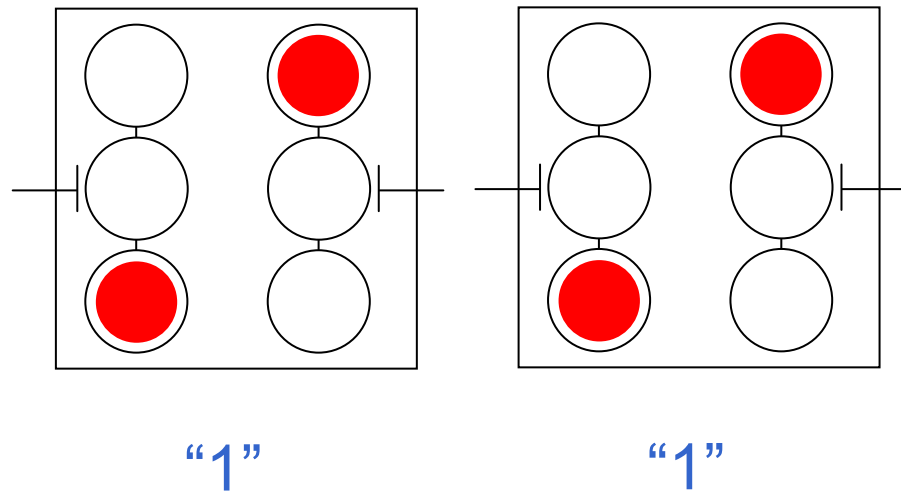
# Quantum-dot cellular automata

Neighboring cells tend to align in the same state.



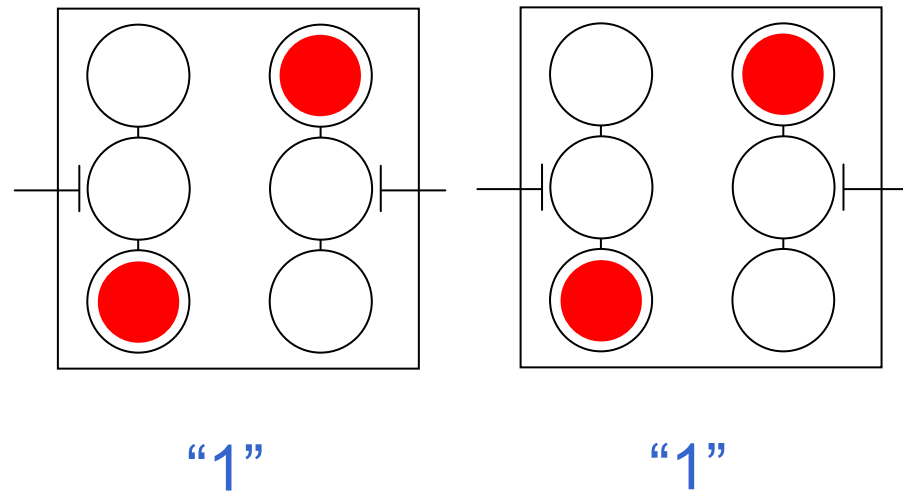
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# Quantum-dot cellular automata

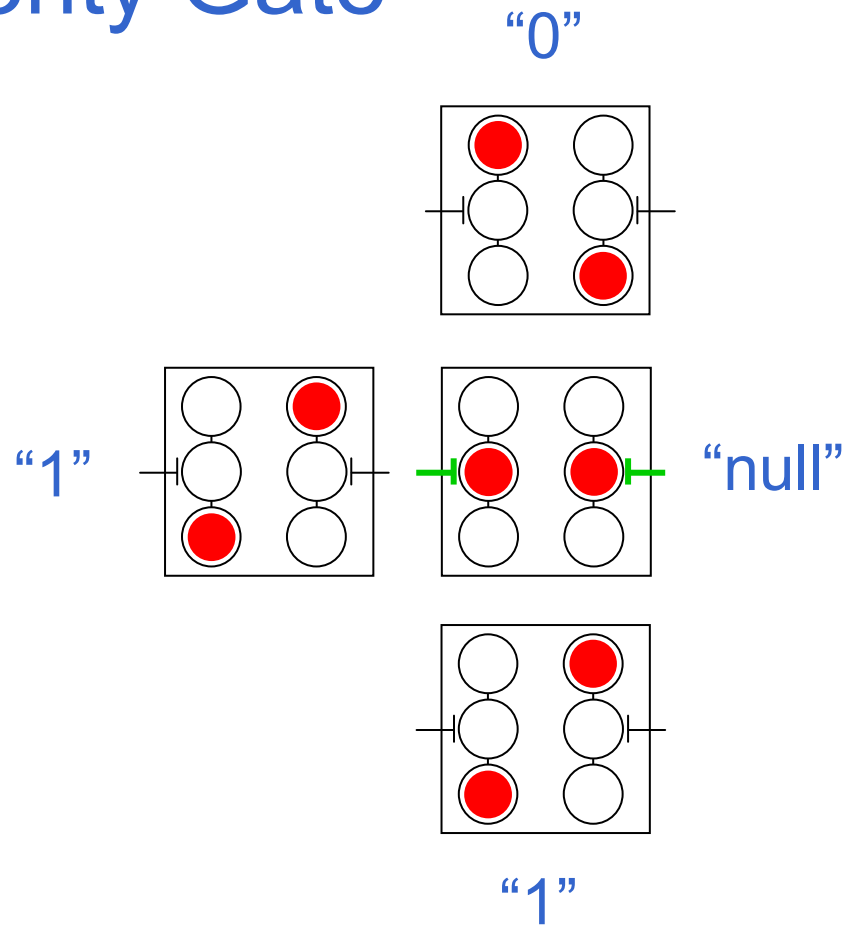
Neighboring cells tend to align in the same state.



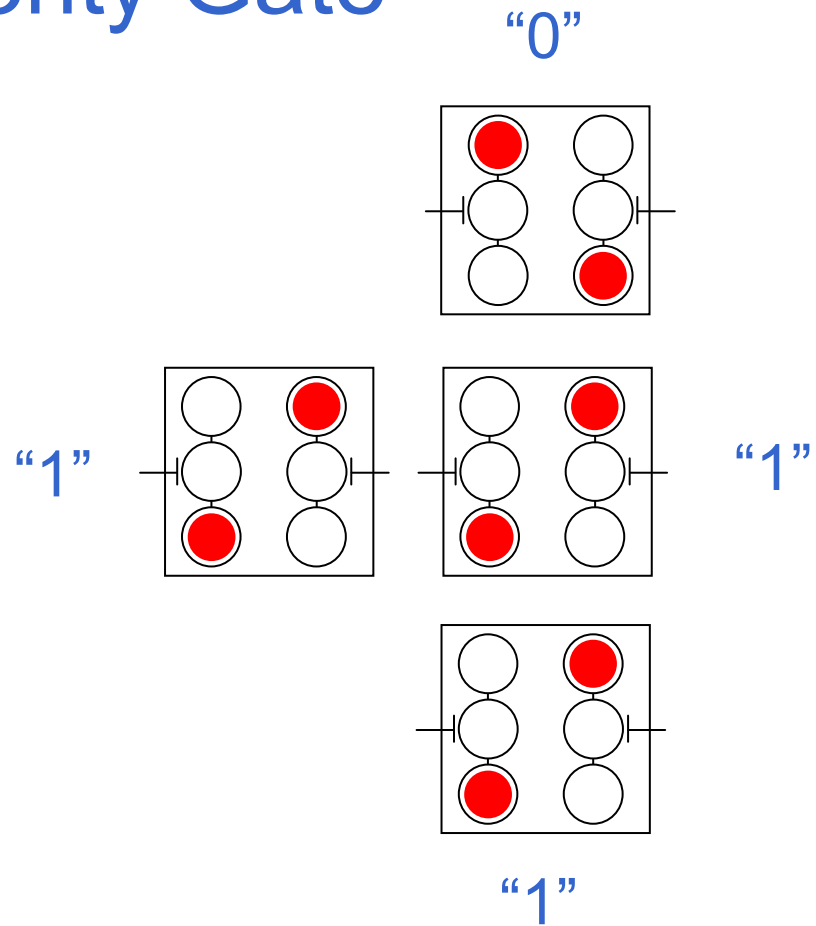
This is the COPY operation.



# Majority Gate

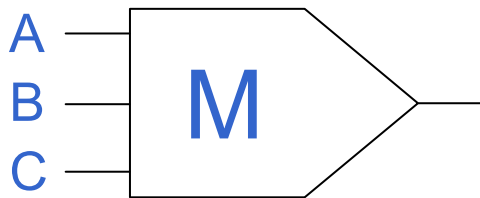


# Majority Gate

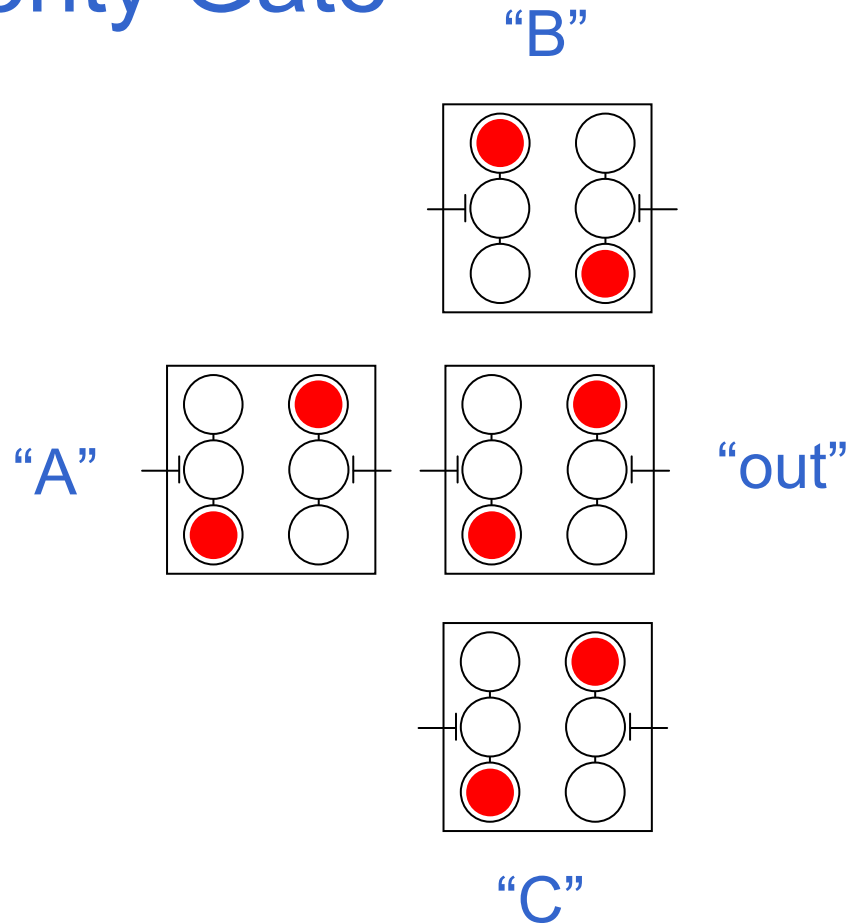




# Majority Gate



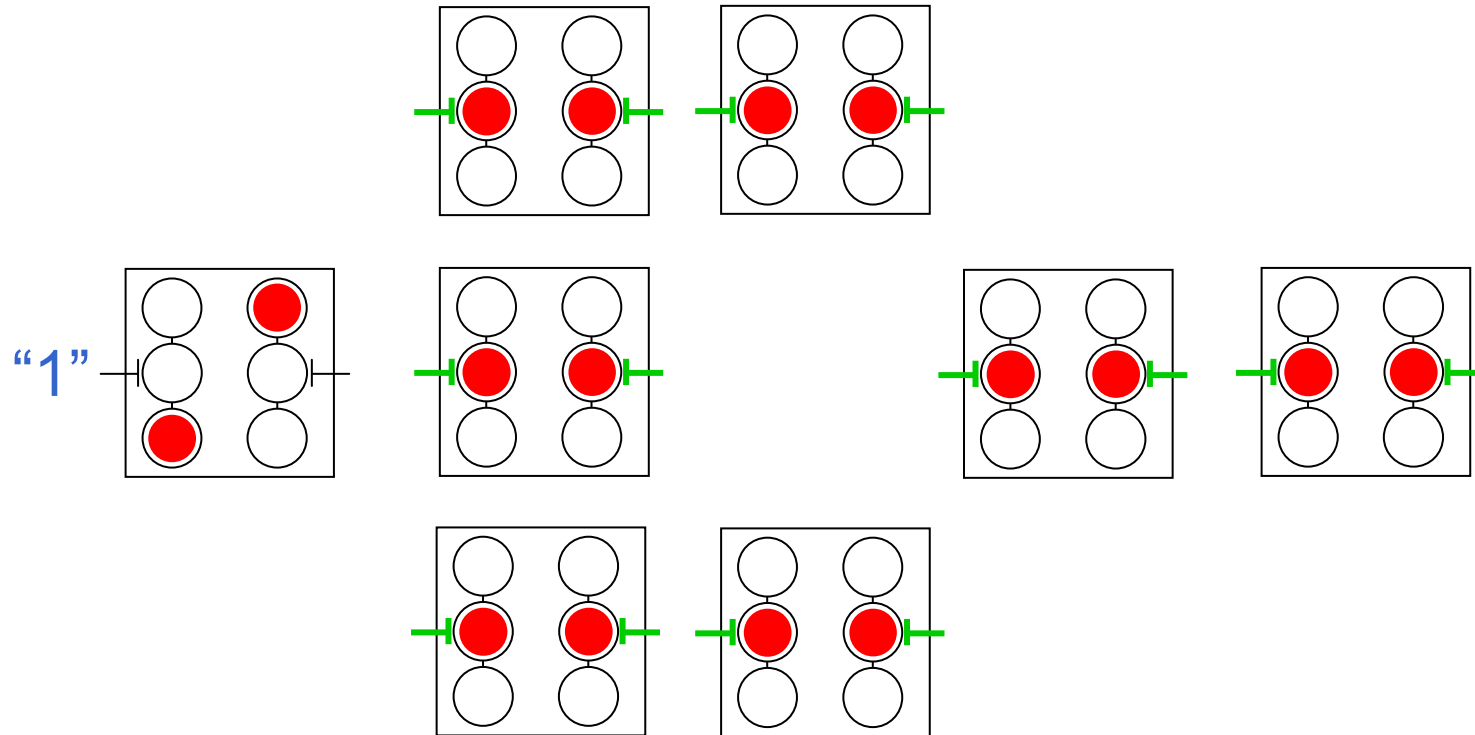
	A	B	C	Output
AND gate	0	0	0	0
	0	0	1	0
	0	1	1	1
OR gate	0	1	0	0
	1	1	0	1
	1	1	1	1
	1	0	1	1
	1	0	0	0



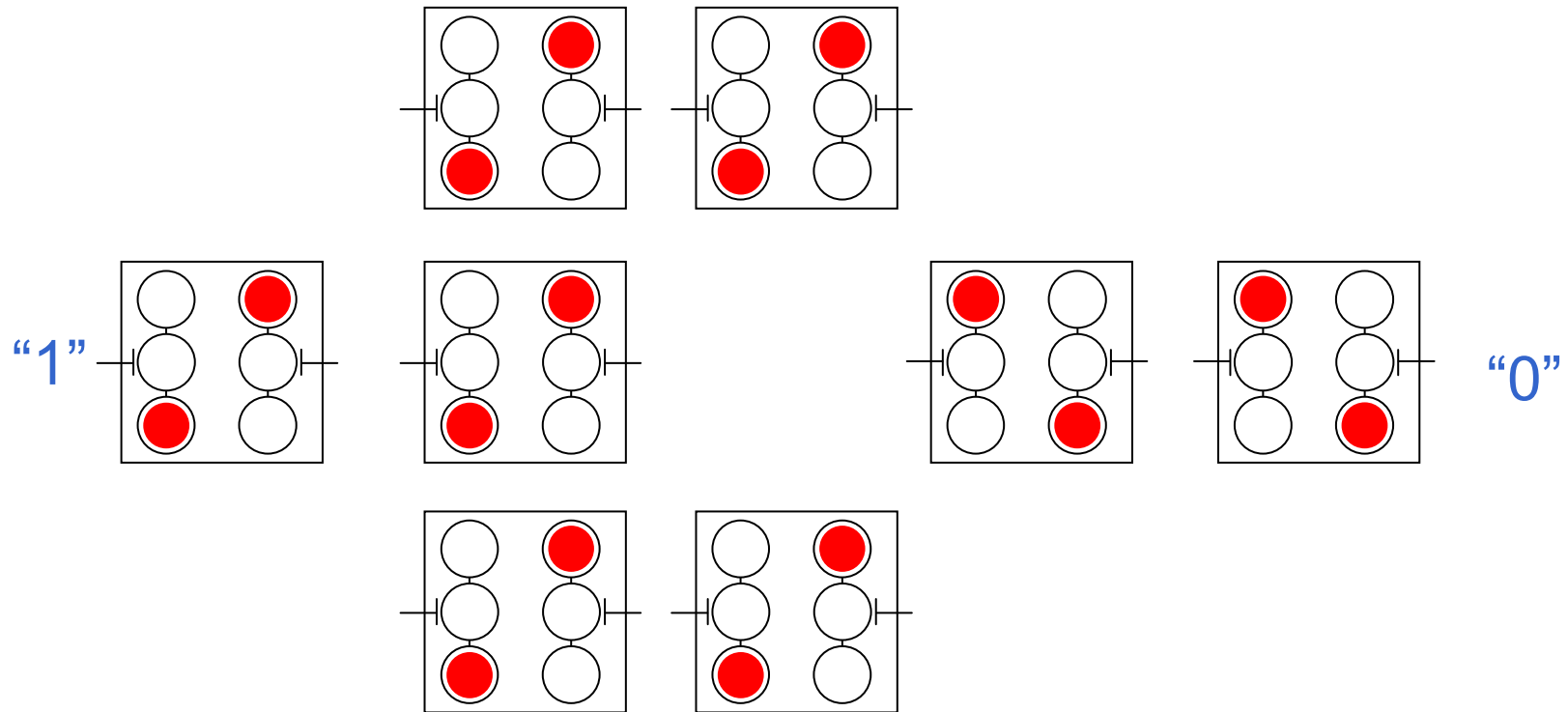
Three input majority gate can function as programmable 2-input AND/OR gate.



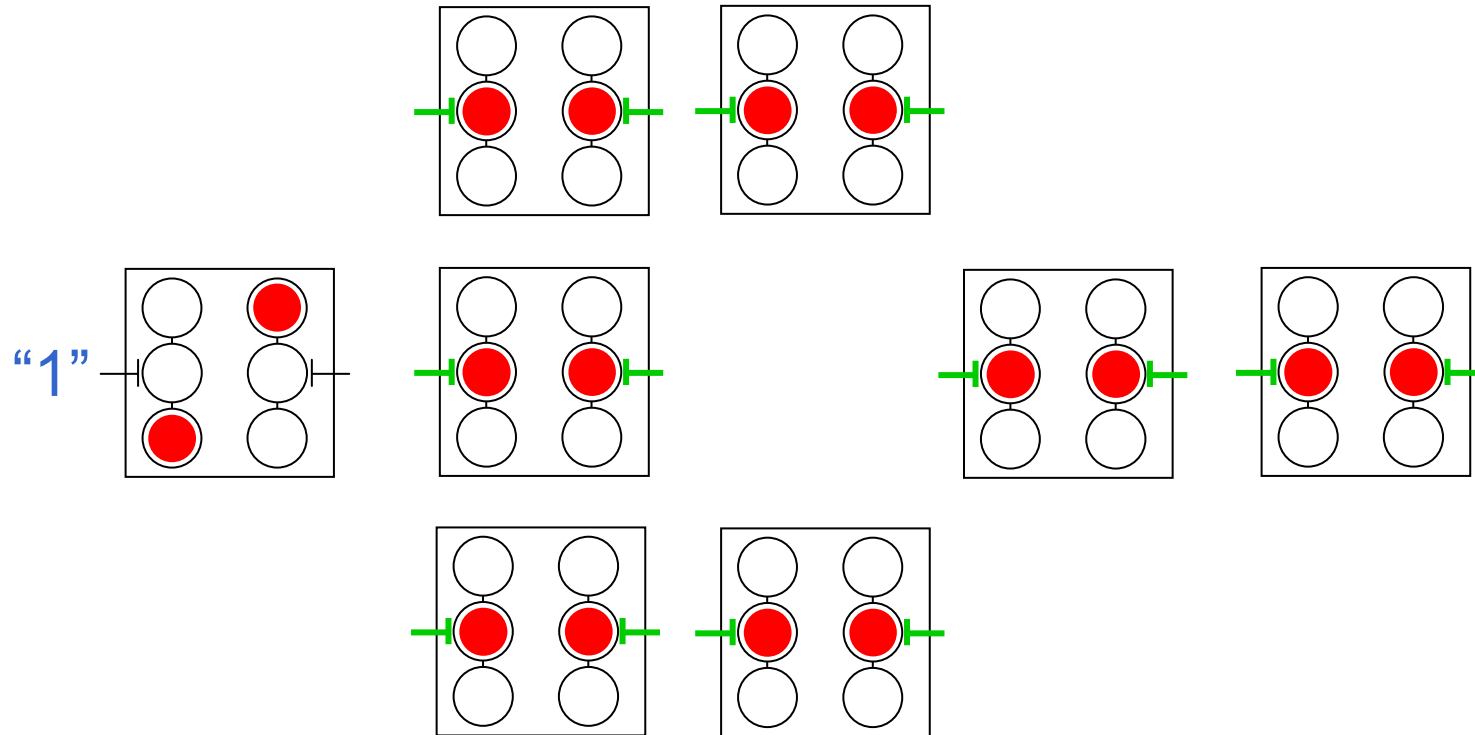
# Inverter Gate



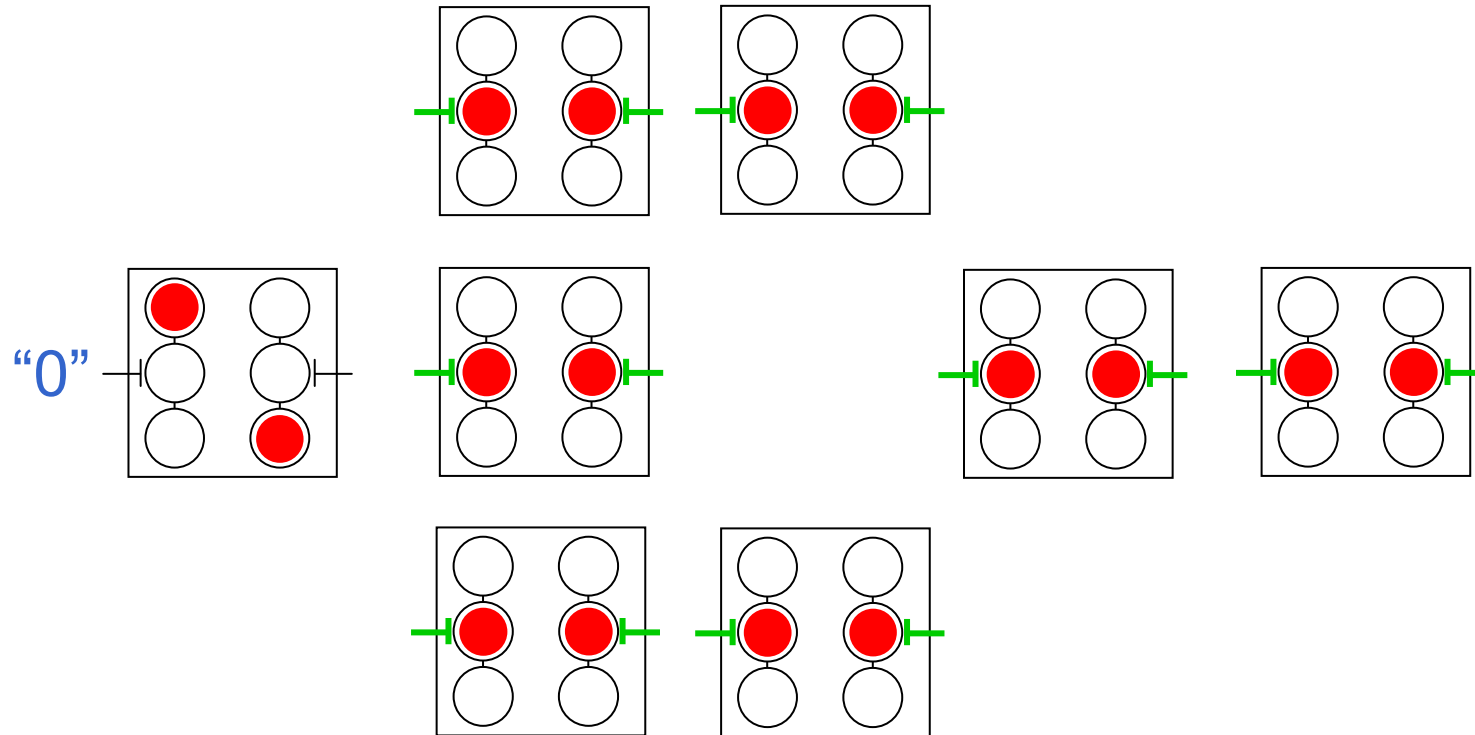
# Inverter Gate



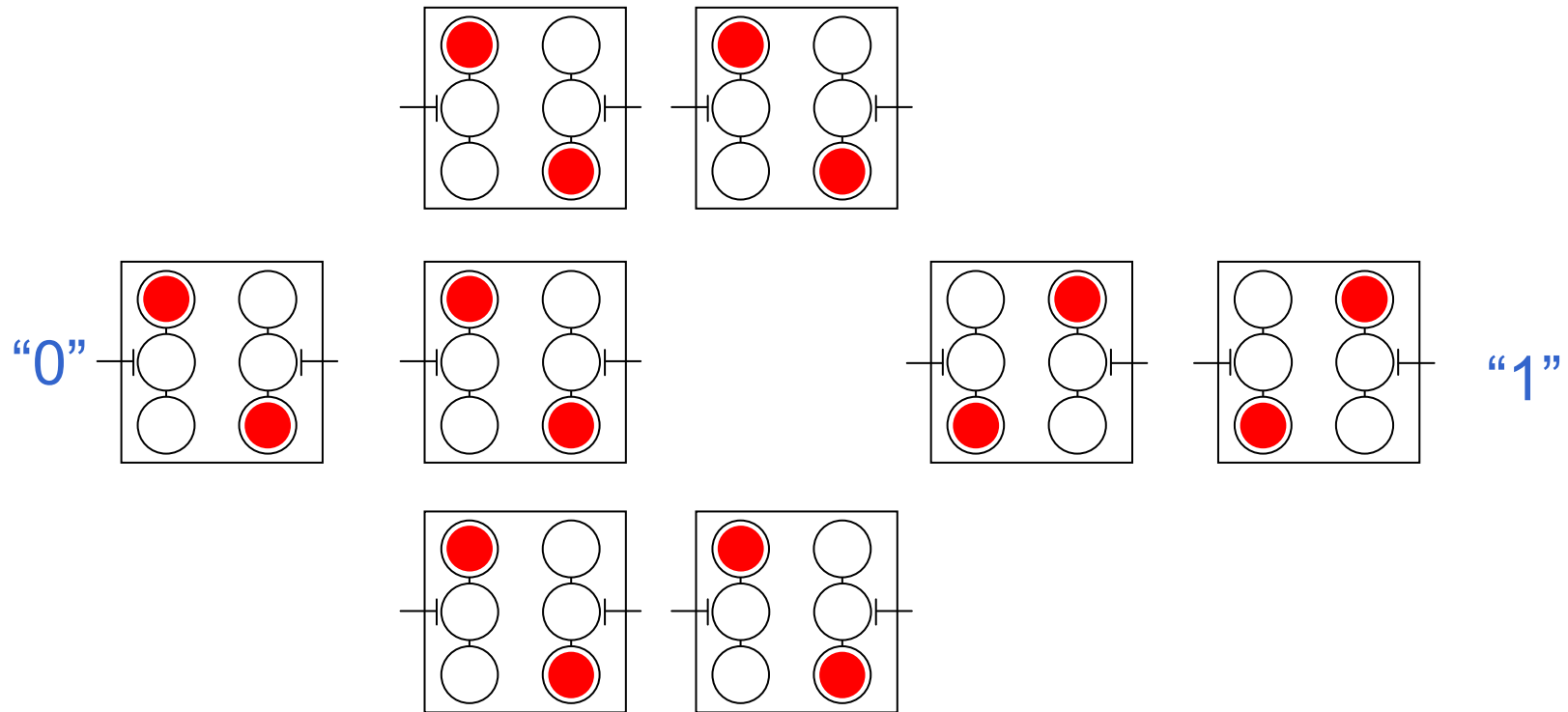
# Inverter Gate



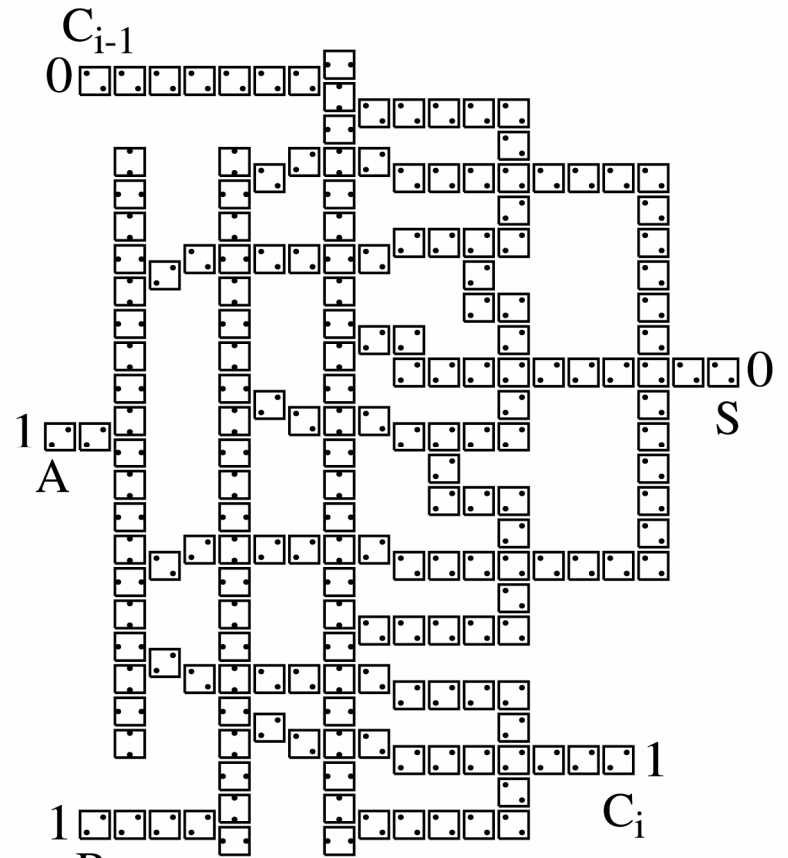
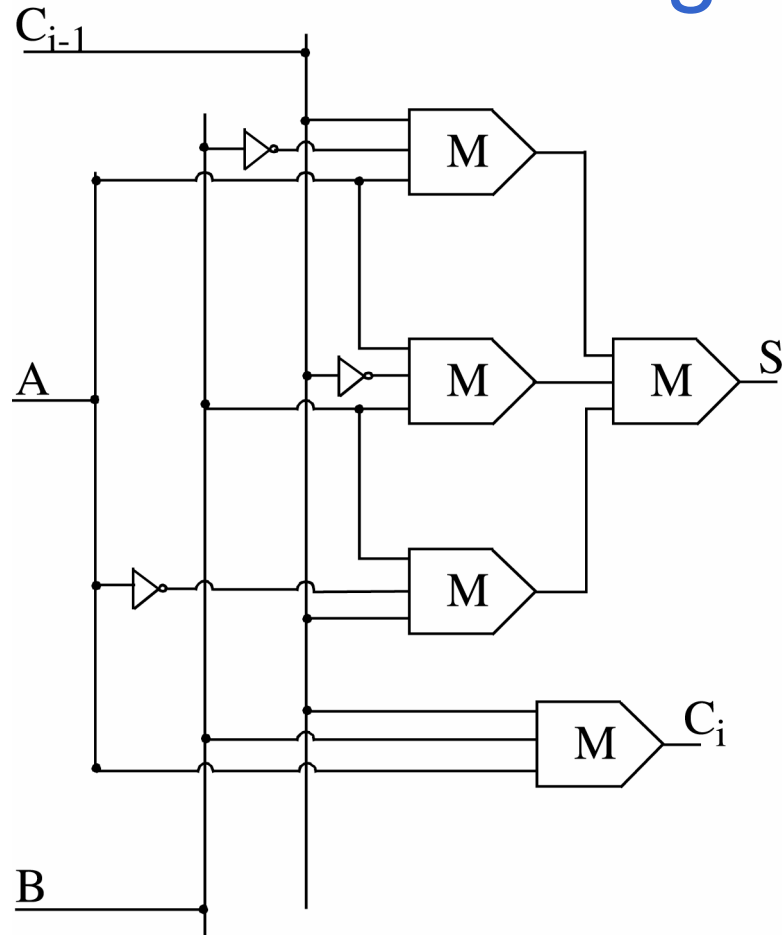
# Inverter Gate



# Inverter Gate



# QCA single-bit full adder

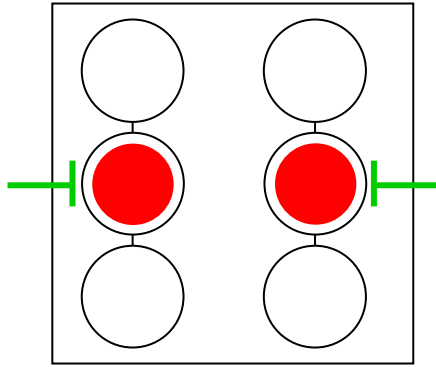


result of SC-HF calculation  
with site model

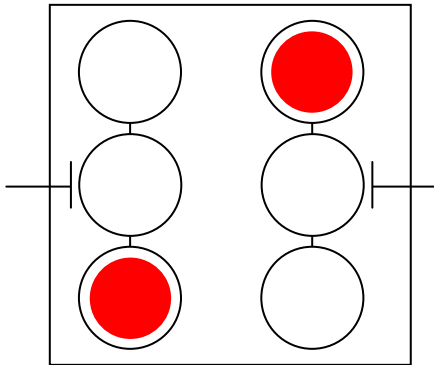
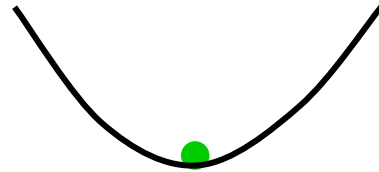
Hierarchical layout and design are possible.



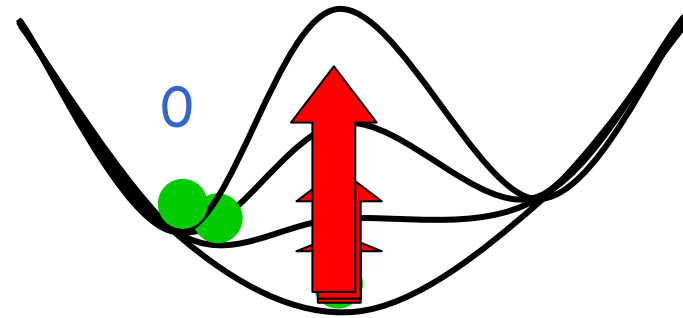
# Adiabatic computing (Landauer)



“null”

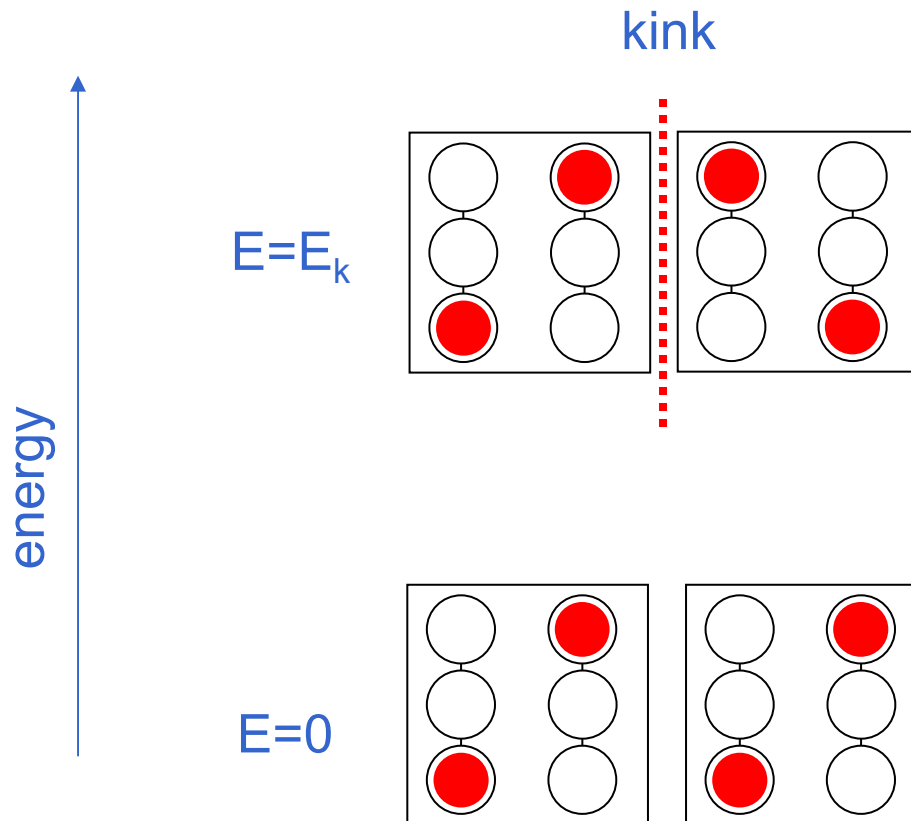


“0”





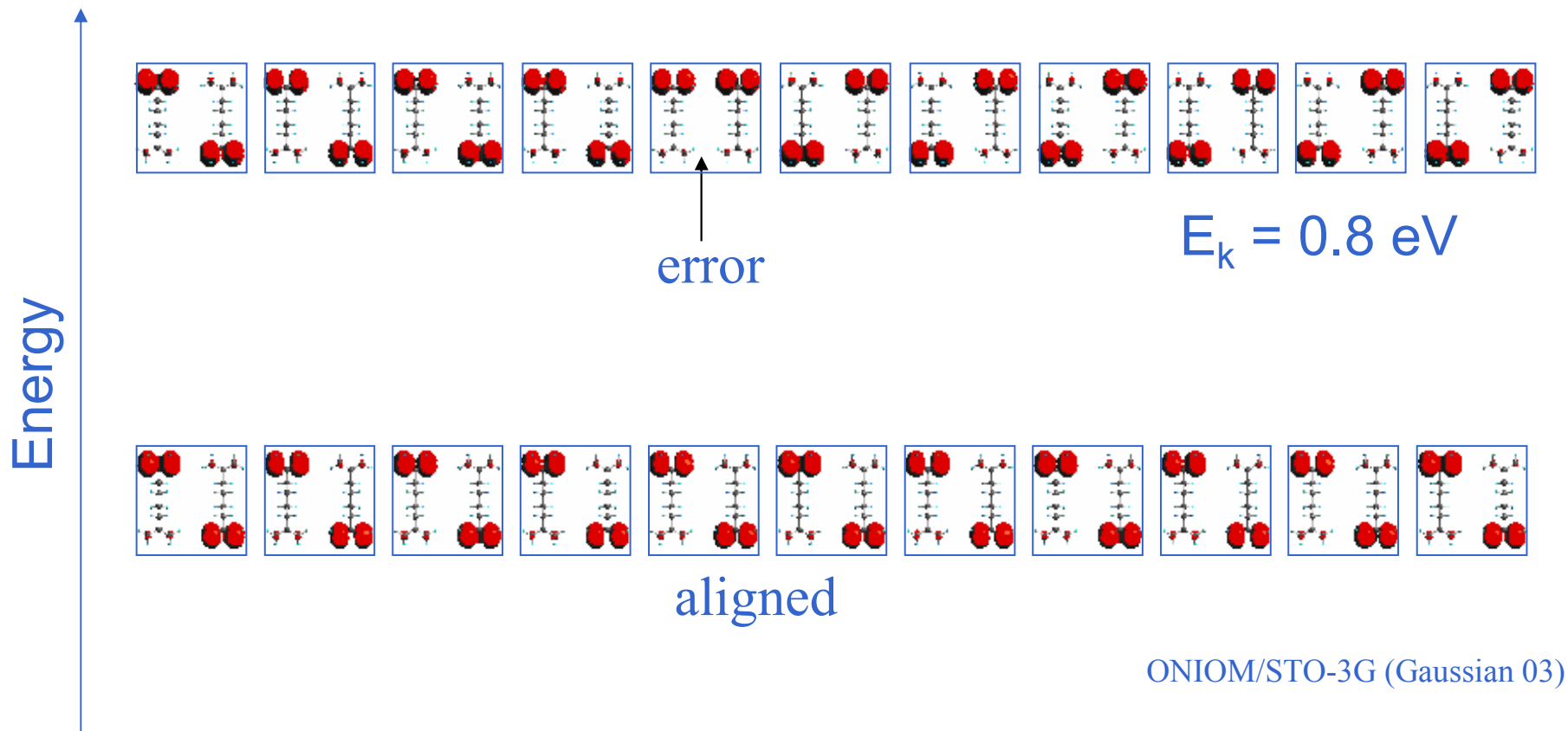
# Characteristic energy



We would like “kink energy”  $E_k > k_B T$ .



# Molecular Wire

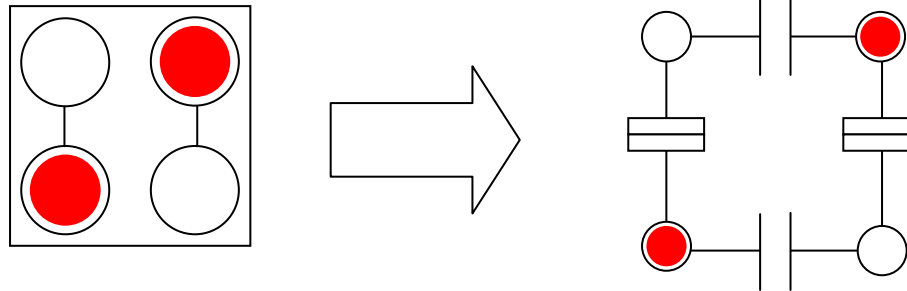


# Outline

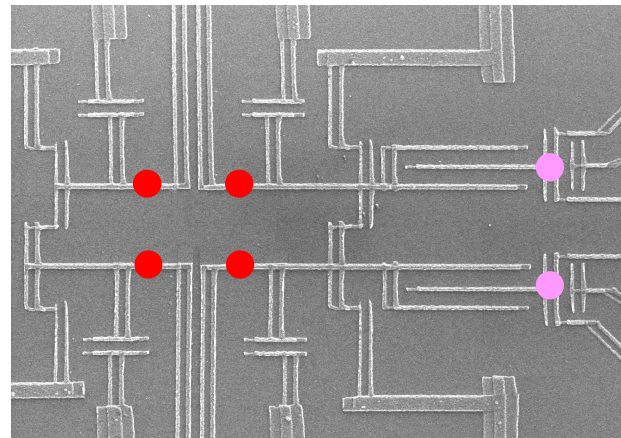
- Introduction
- QCA paradigm
- Implementations
  - Metal-dot QCA
  - Molecular QCA
- Energy flow in QCA
  - Power gain
  - Power dissipation and erasure
  - Bennett clocking



# QCA devices exist



## Metal-dot QCA implementation



Al/AIO<sub>x</sub> on  
SiO<sub>2</sub>

electrometers

“dot” = metal island

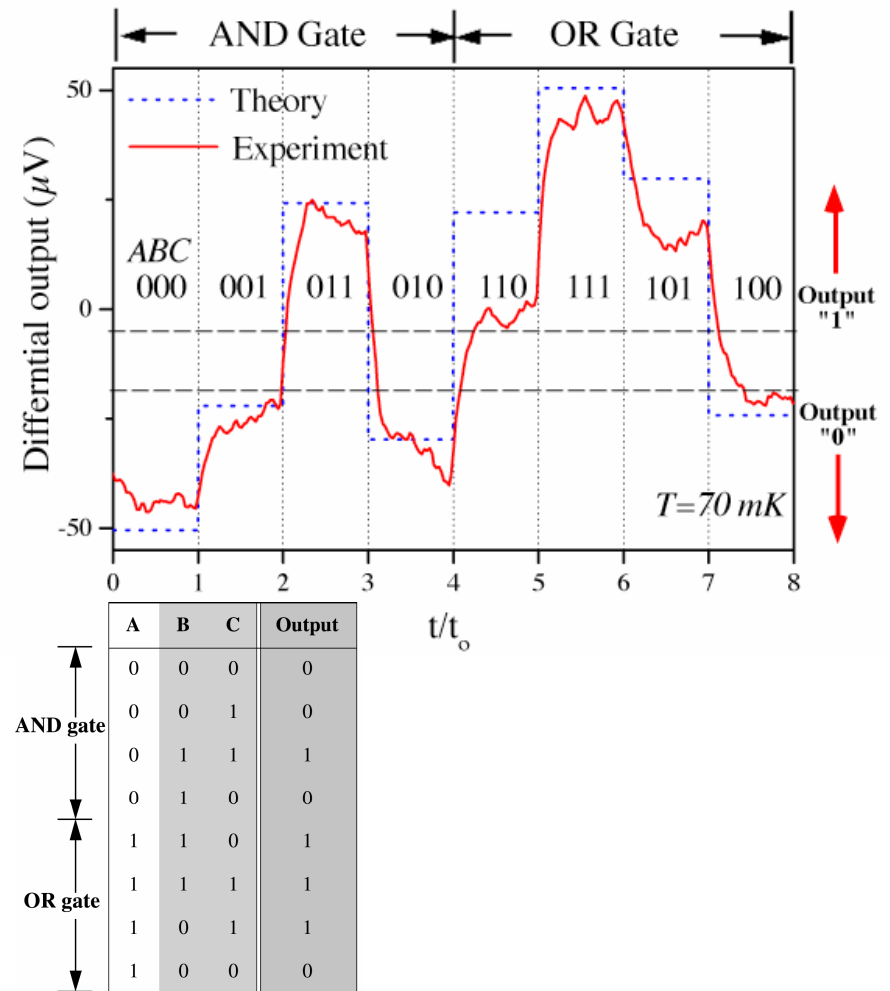
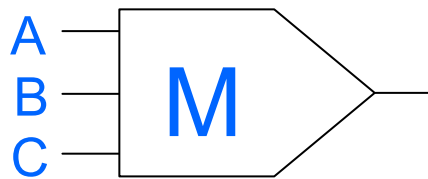
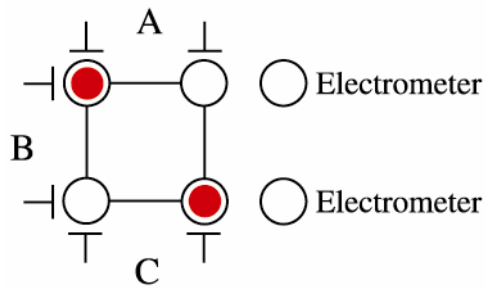
70 mK

*Greg Snider, Alexei Orlov, and Gary Bernstein*



# Metal-dot QCA cells and devices

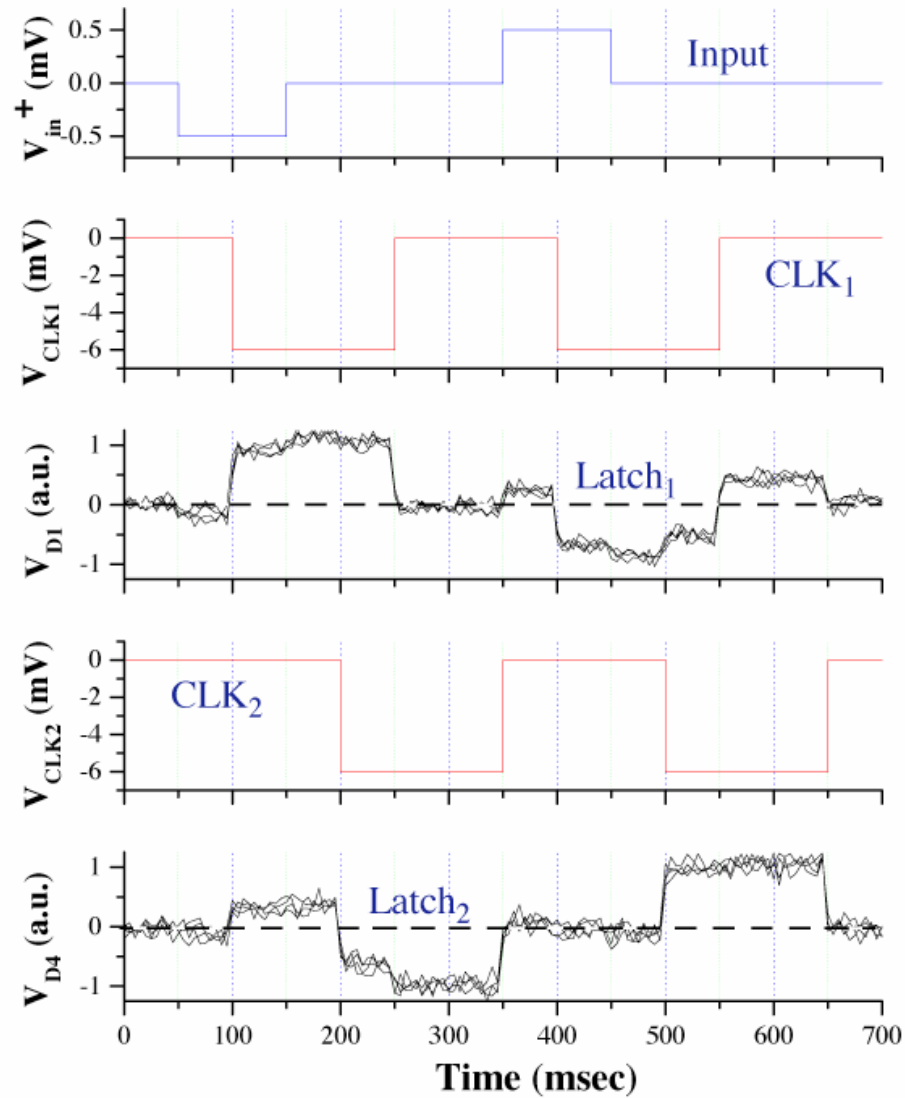
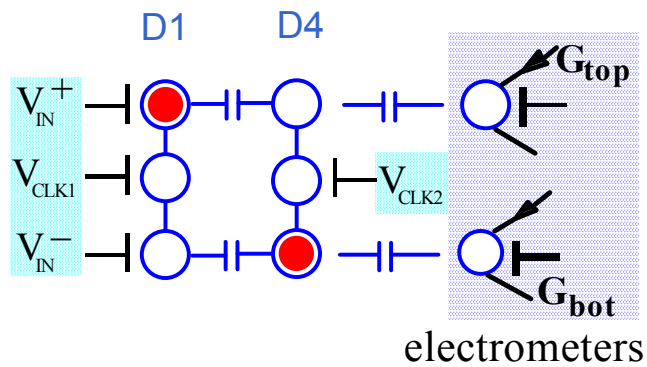
- Majority Gate



Amlani, A. Orlov, G. Toth, G. H. Bernstein, C. S. Lent, G. L. Snider,  
*Science* **284**, pp. 289-291 (1999).

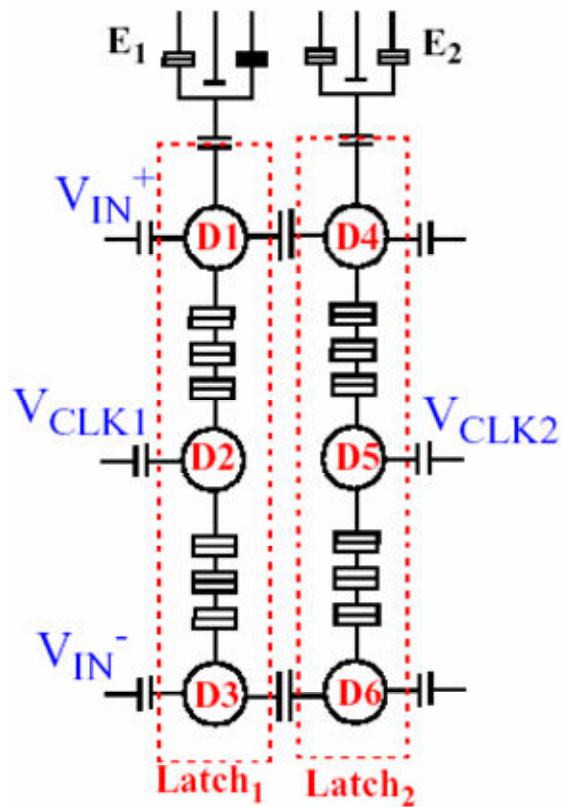


# QCA Shift Register

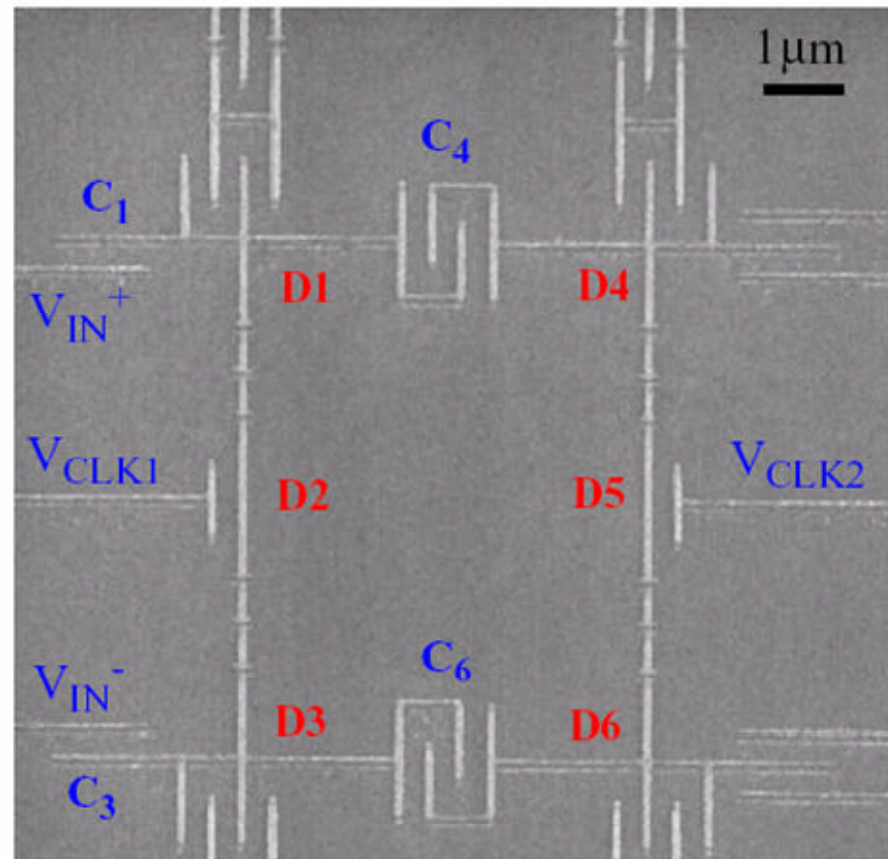


# QCA Shift Register

Schematic Diagram



SEM Micrograph



# Metal-dot QCA devices exist

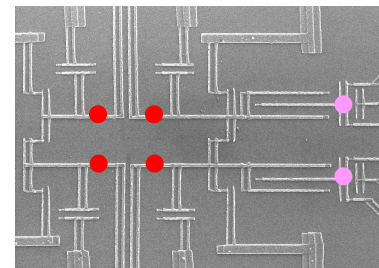
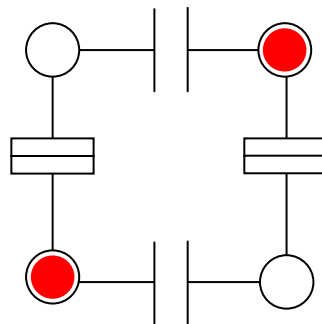
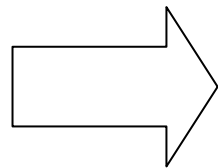
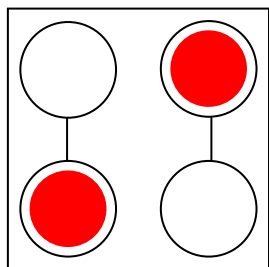
- Single electron analogue of molecular QCA
- Gates and circuits:
  - Wires
  - Shift registers
  - Fan-out
  - Power gain demonstrated
  - AND, OR, Majority gates
- Work underway to raise operating temperatures





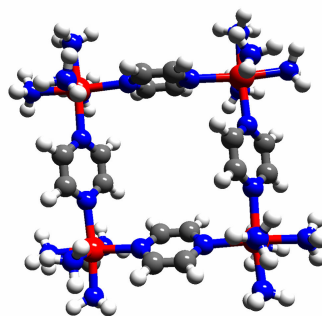
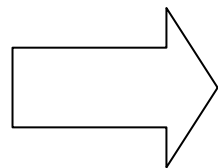
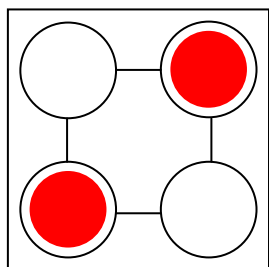
# From metal-dot to molecular QCA

Metal tunnel junctions



“dot” = metal island

70 mK



“dot” = redox center

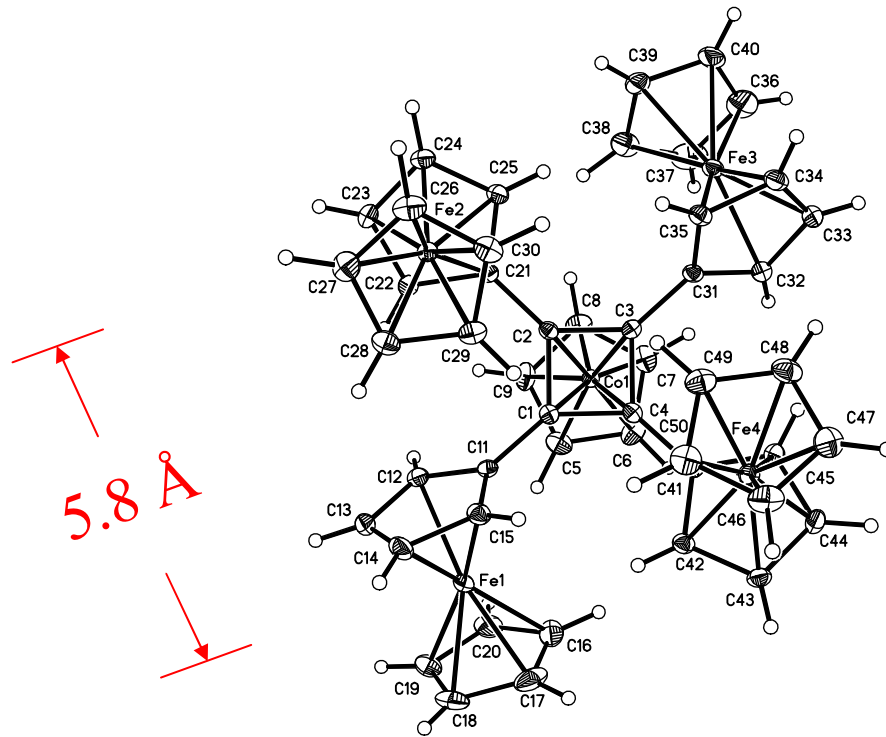
Mixed valence compounds

room temperature+

Key strategy: use *nonbonding* orbitals ( $\pi$  or d) to act as dots.



# 4-dot molecule

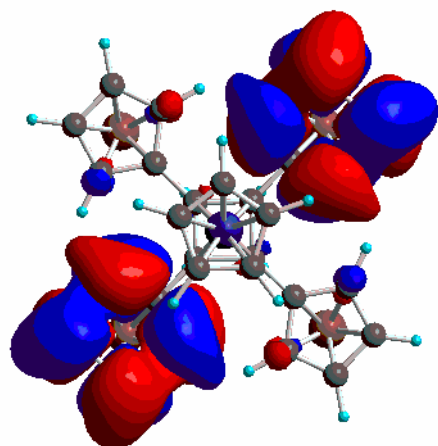


Fehlner *et al*  
(Notre Dame chemistry group)  
*Journal of American Chemical Society*  
125:7522, 2003

Each ferrocene acts as a quantum dot, the Co group connects 4 dots.

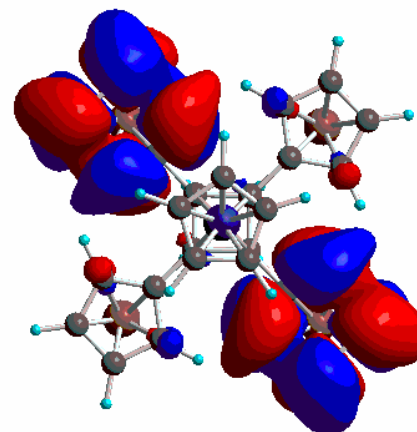


# Bistable configurations



“0”

Fehlner *et al*  
(Notre Dame chemistry group)  
*Journal of American Chemical Society*  
125:7522, 2003

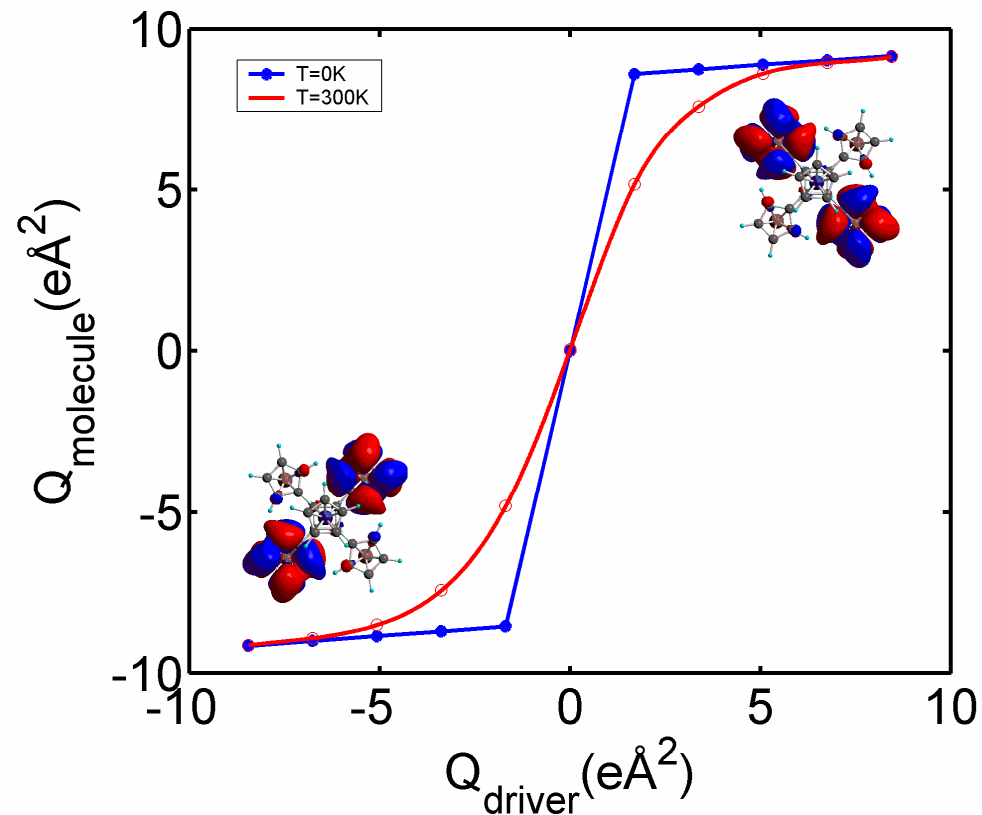


“1”

Guassian-98 UHF/STO-3G/LANL2DZ



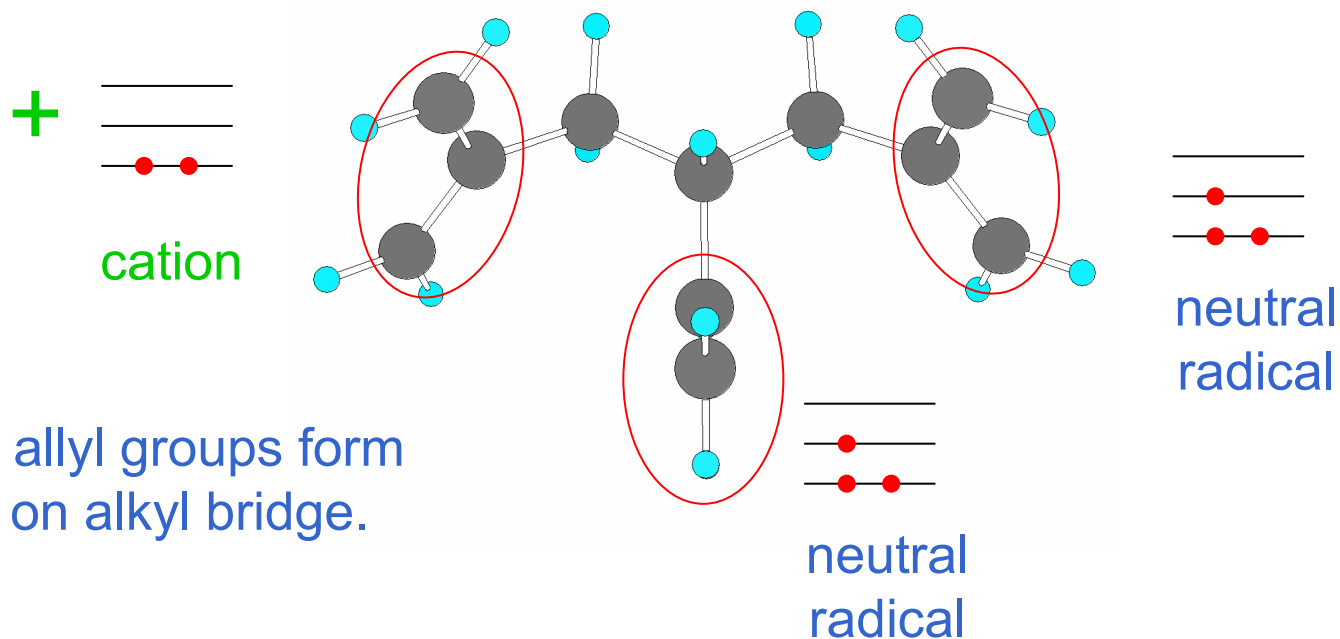
# Switching molecule by a neighboring molecule



Coulomb interaction is sufficient to couple molecular states.



# Molecular 3-dot cell

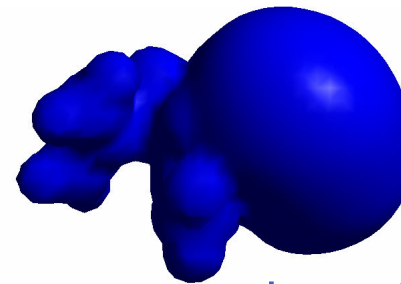
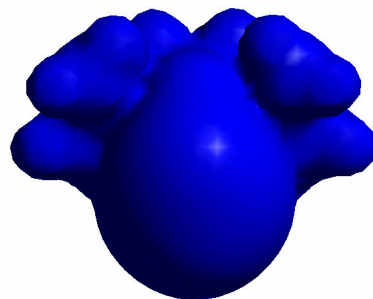
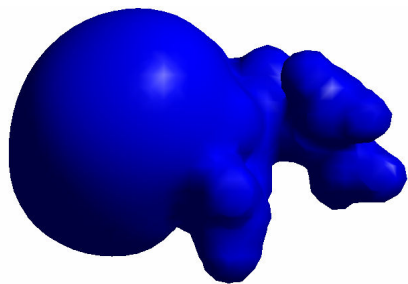


Three allyl groups form  
“dots” on alkyl bridge.

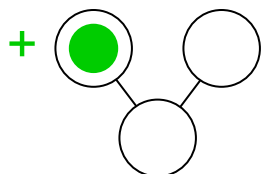
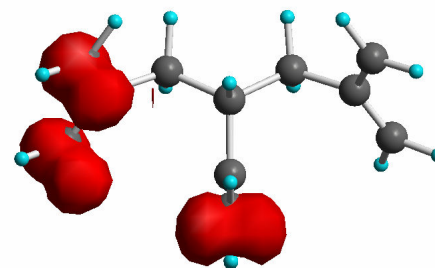
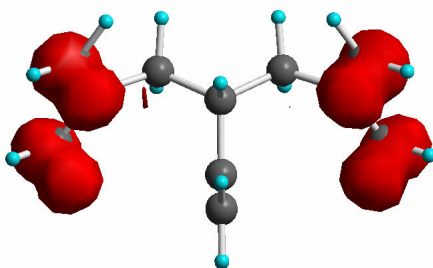
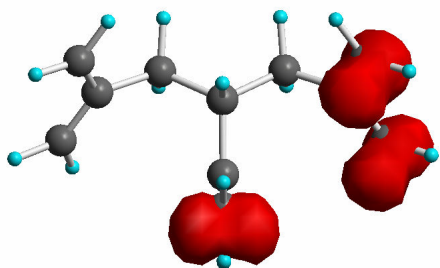
For the molecular cation, a hole occupies one of three dots.



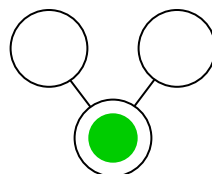
# Charge configuration represents bit



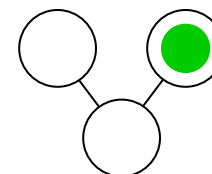
isopotential  
surfaces



“0”



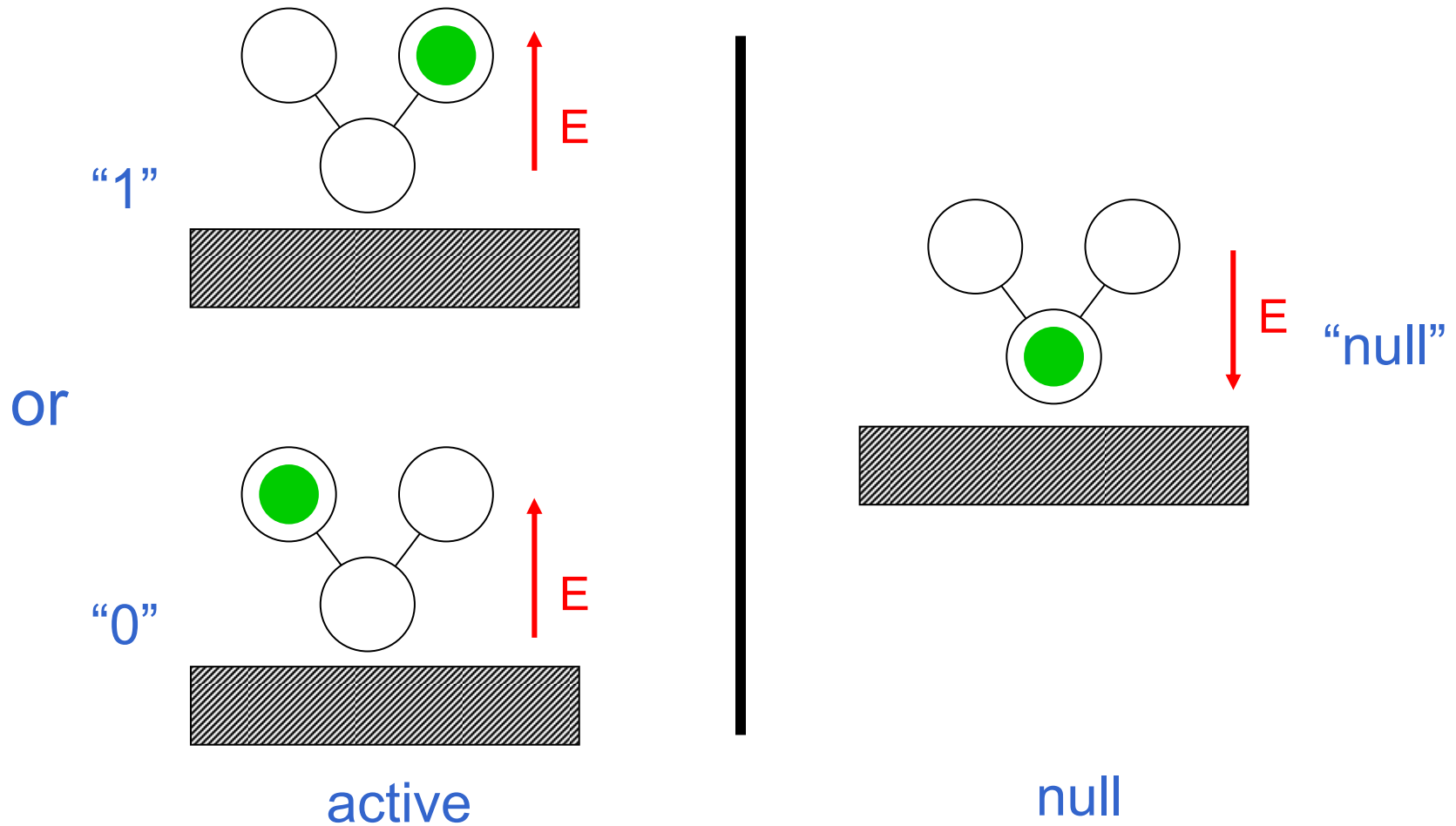
“null”



“1”



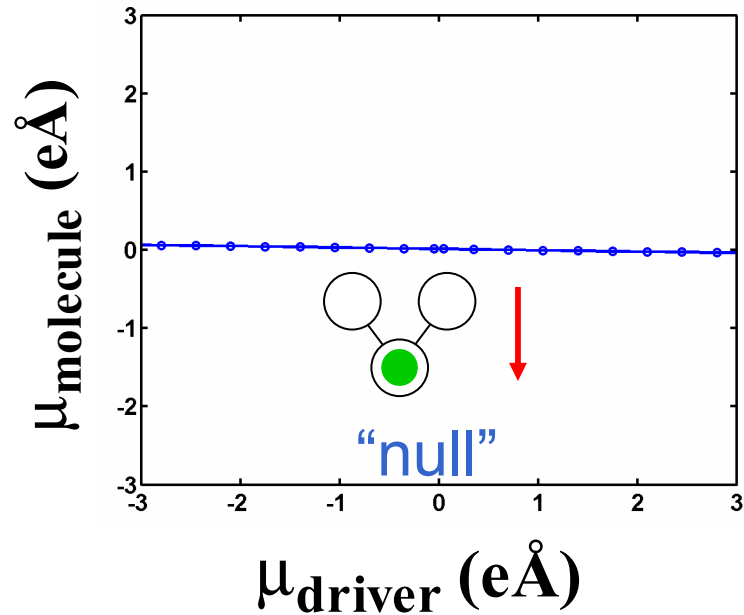
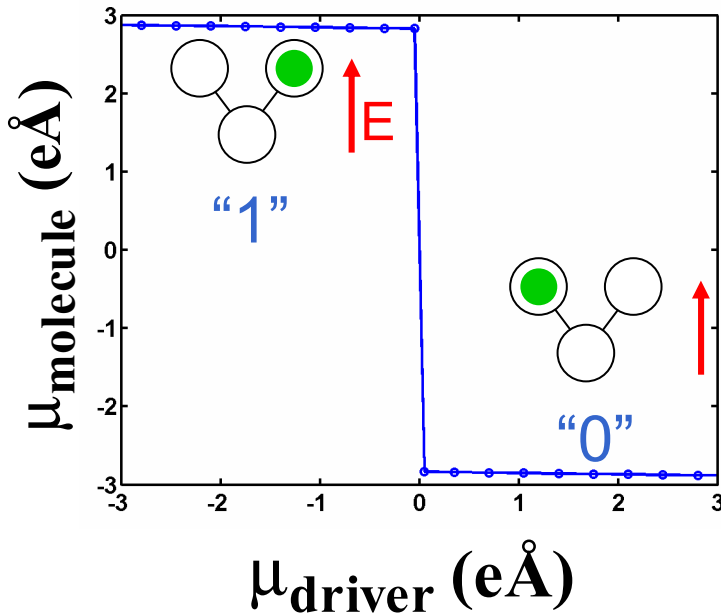
# Clocking field



Use local electric field to switch molecule between active and null states.



# Clocking field alters response function



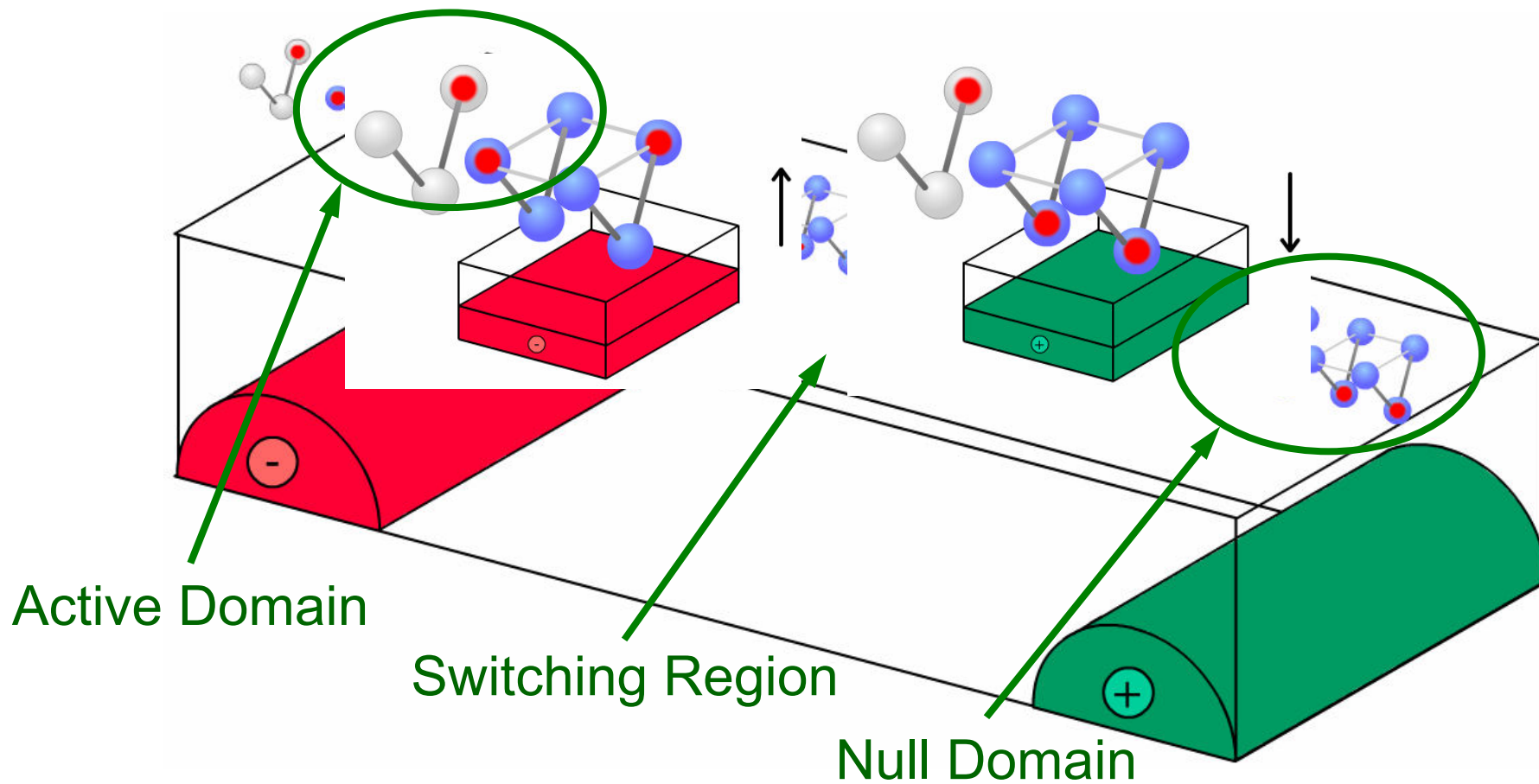
- Clocking field positive (or zero)
- Positive charge in top dots
- Cell is **active** – nonlinear response to input

- Clocking field negative
- Positive charge in bottom dot
- Cell is **inactive** – no response to input





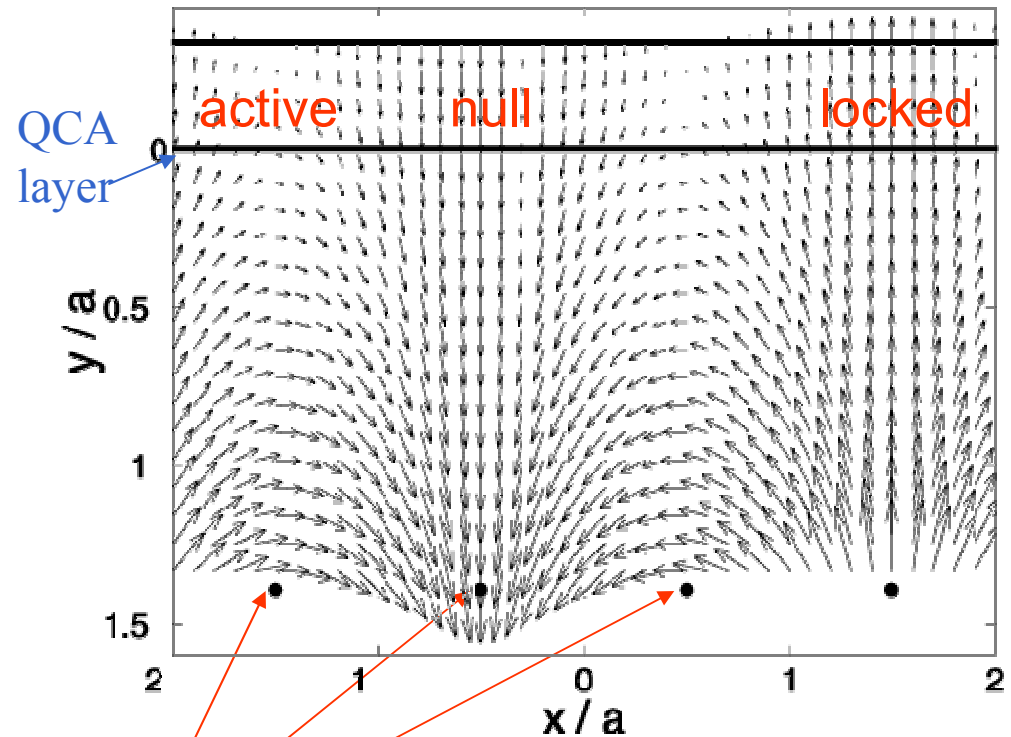
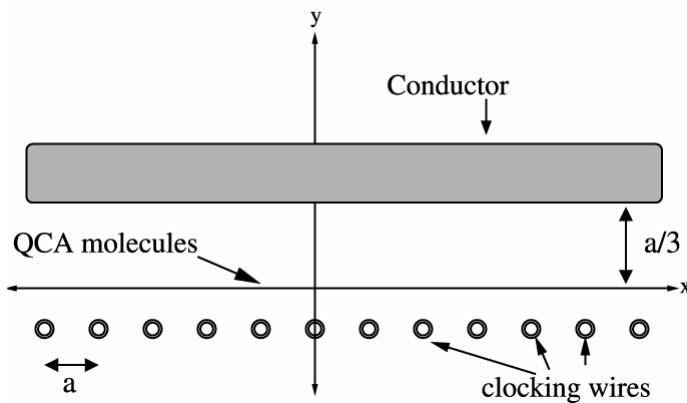
# Clocked Molecular QCA



No current leads. No need to contact individual molecules.



# Molecular clocking



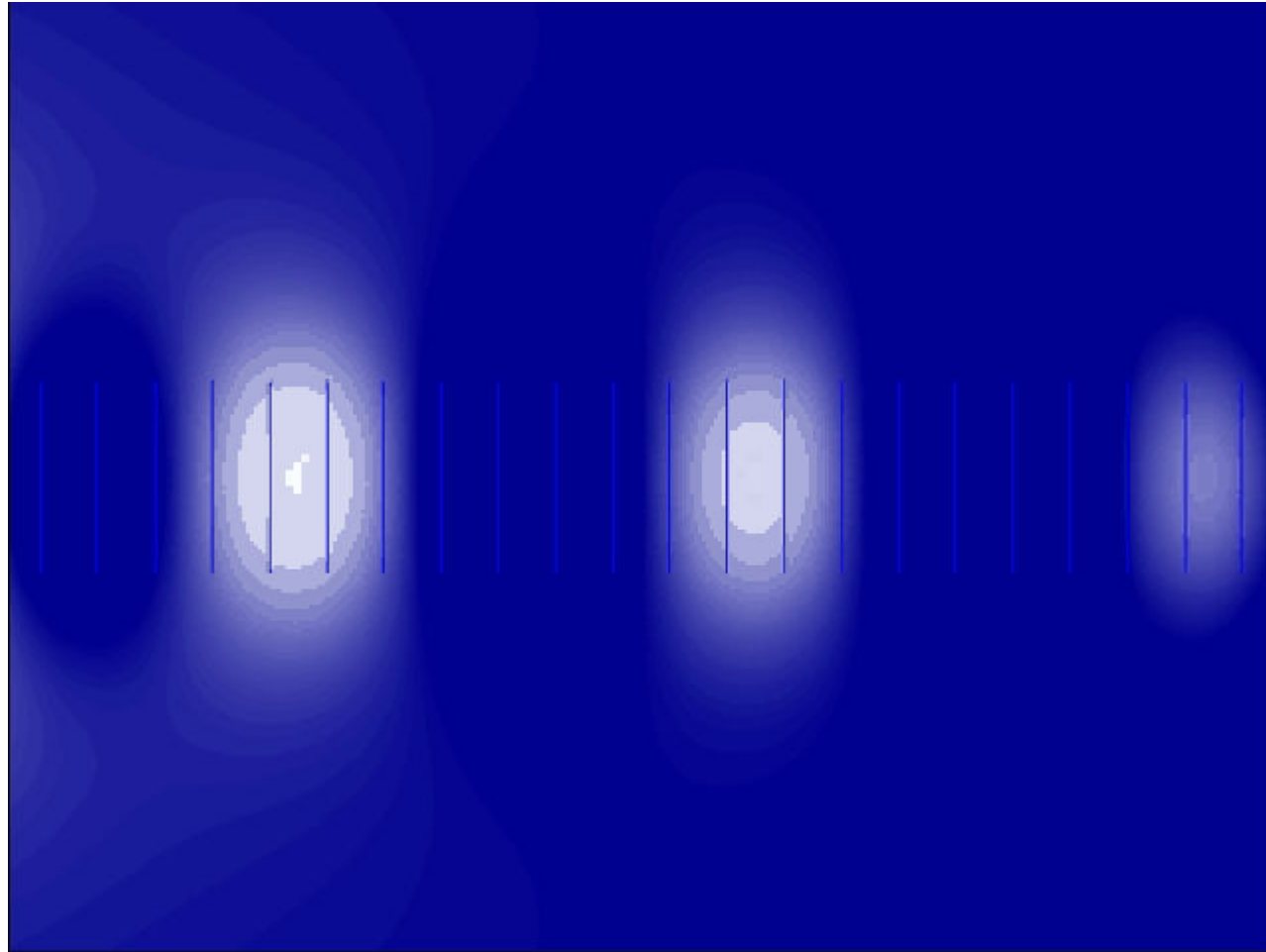
*Hennessey and Lent, JVST (2001)*

Clocking field is provided by **buried wire electrodes** (CMOS controlled).

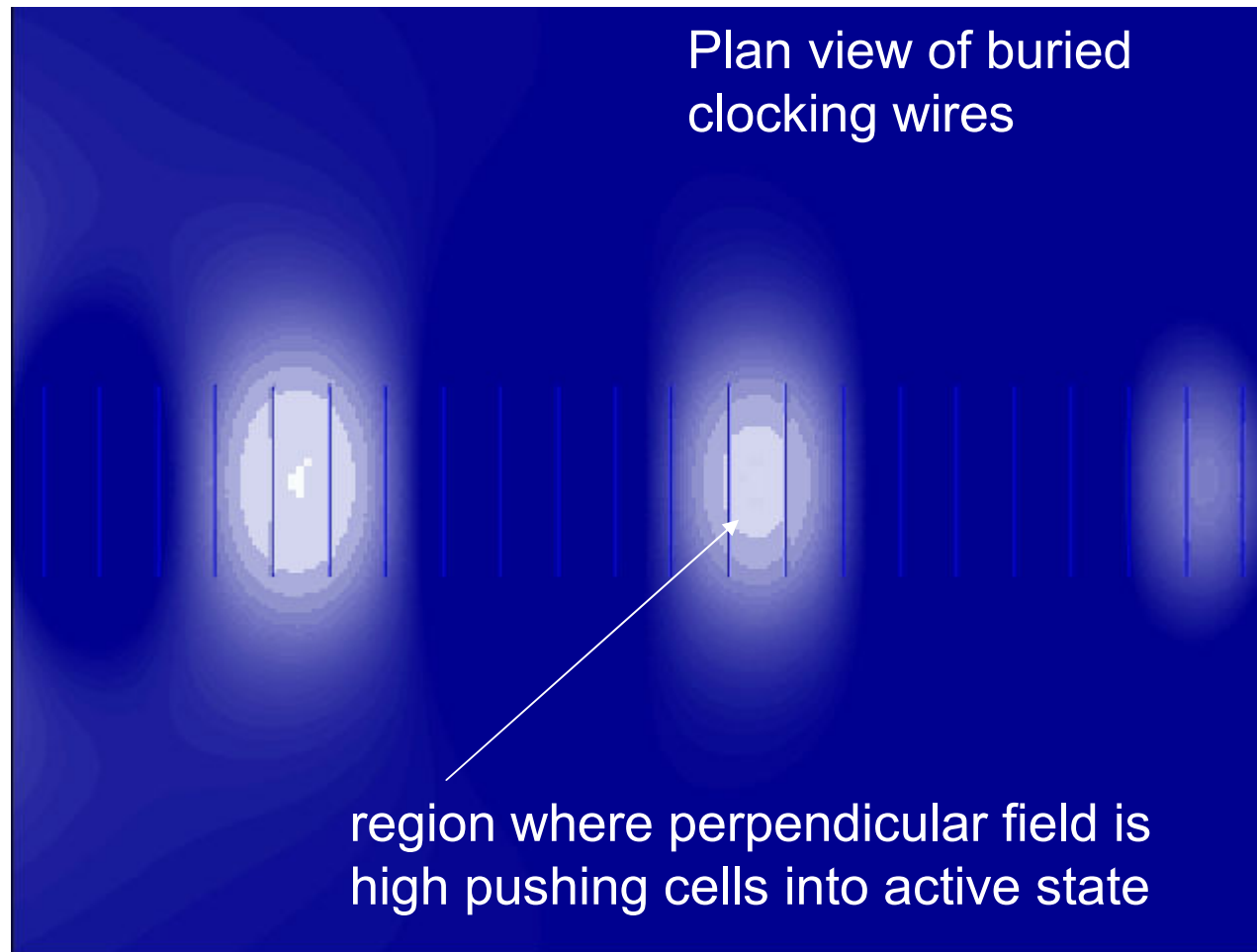
Wire sizes can be 10-100 times larger than molecules.



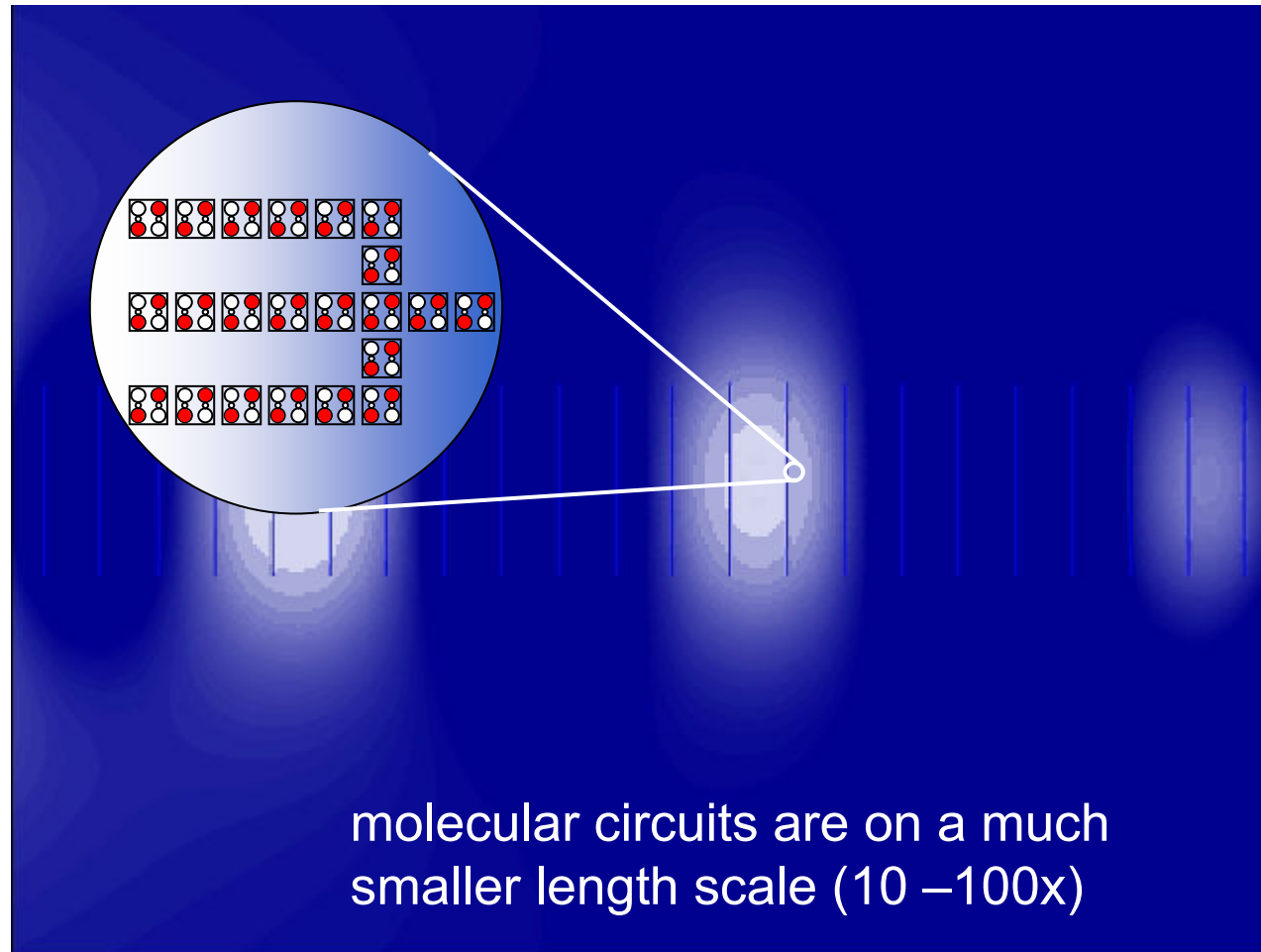
# Clocking field: linear motion



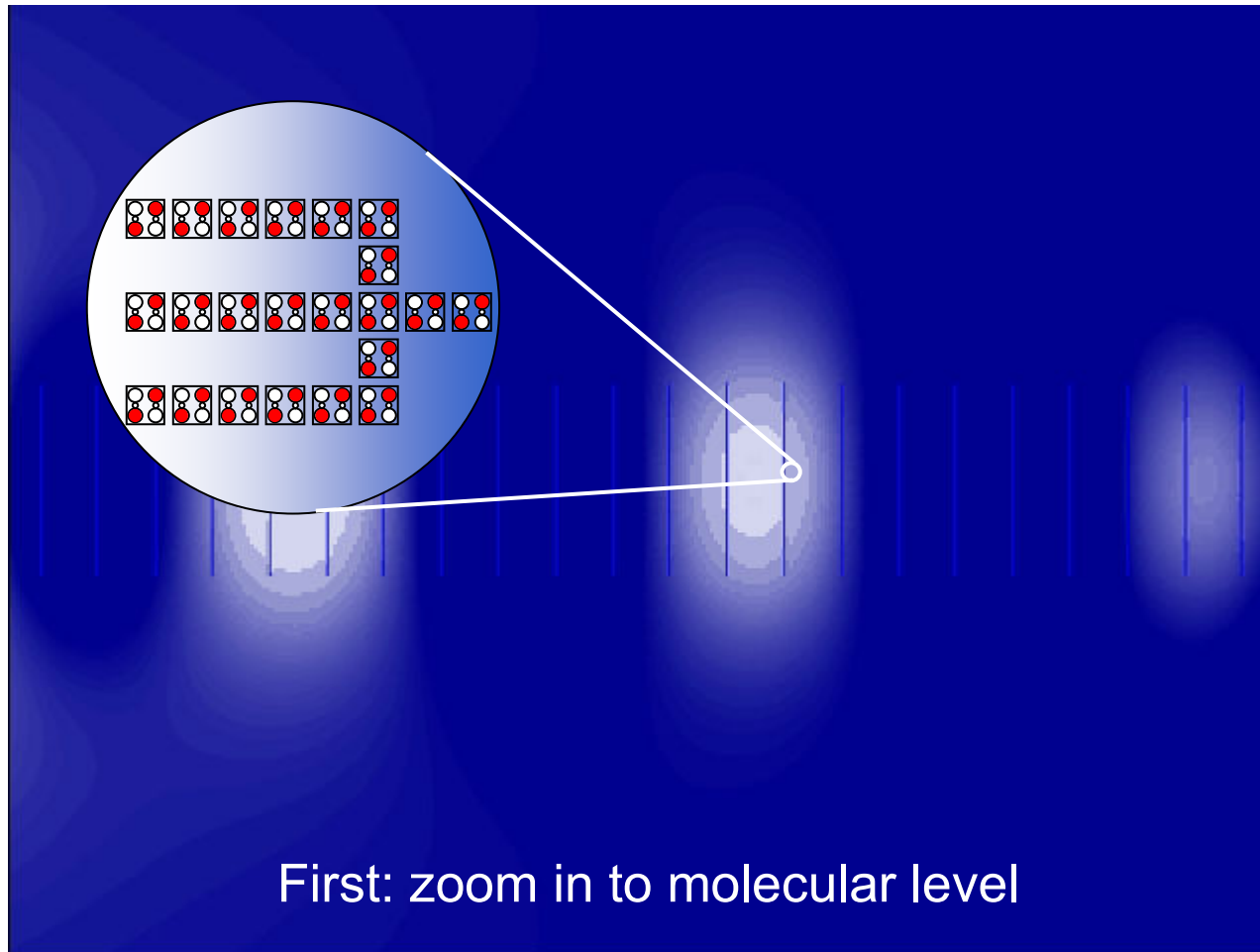
# Molecular circuits and clocking wires



# Molecular circuits and clocking wires



# Molecular circuits and clocking wires



# Field-clocking of QCA wire: shift-register



# Computational wave: majority gate





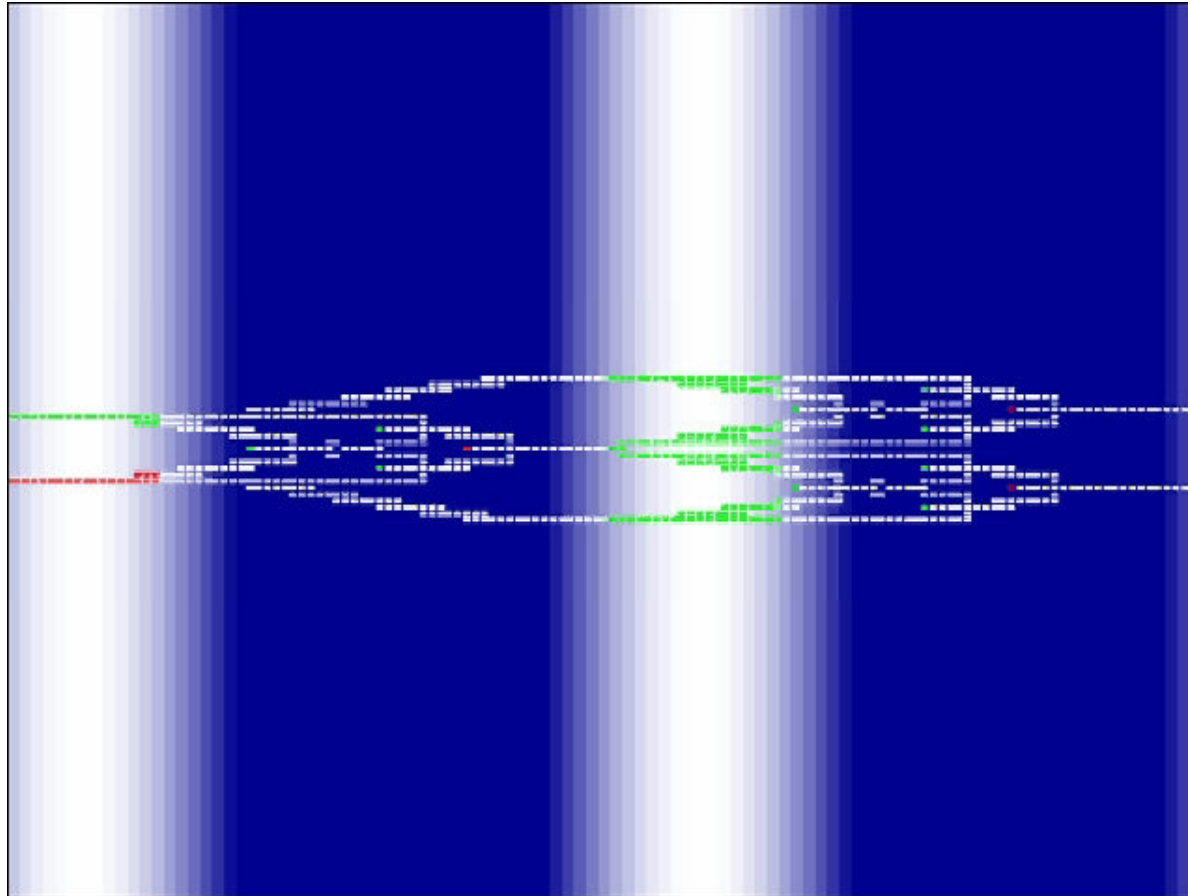
# Computational wave: adder back-end



# XOR Gate



# Permuter

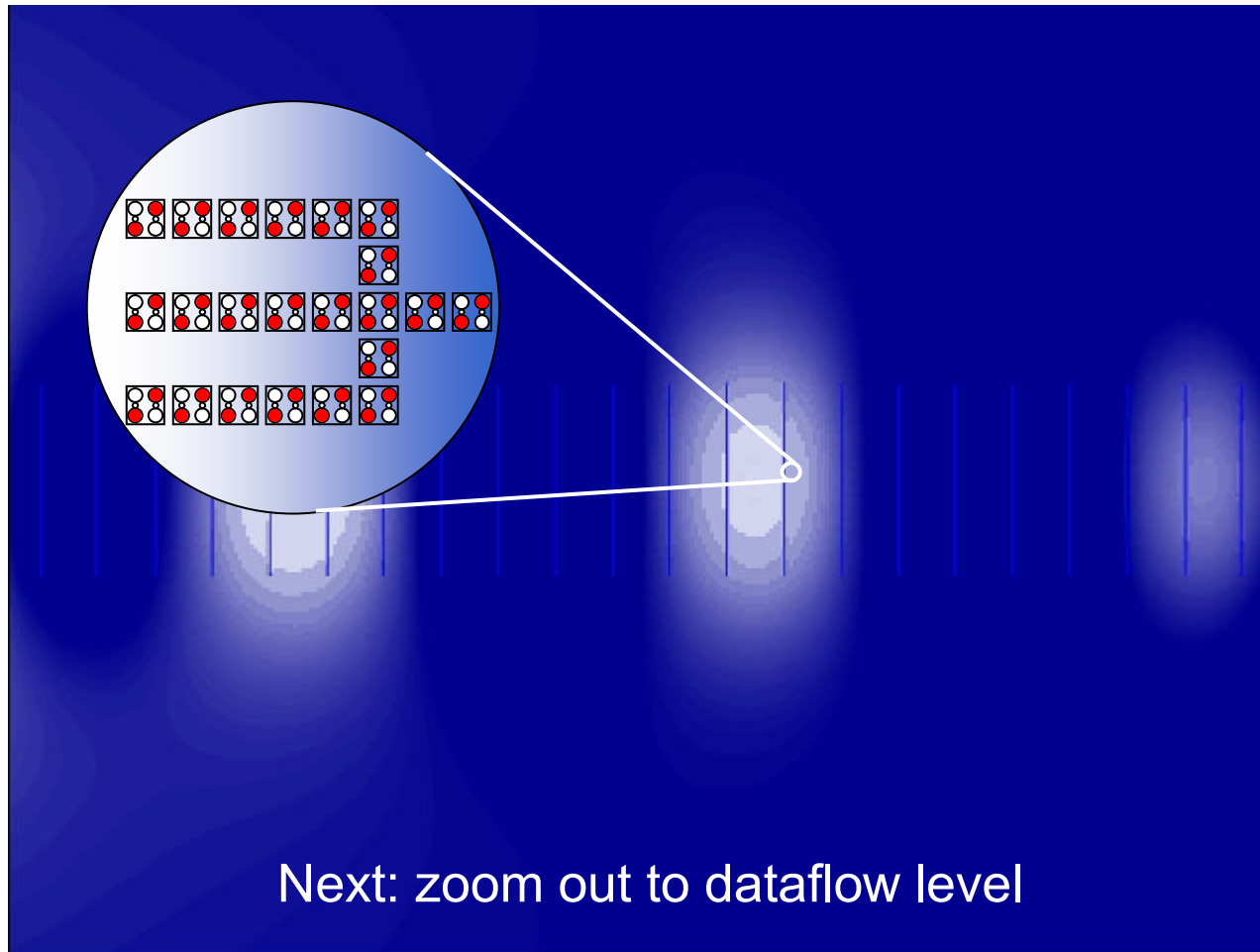


# Wider QCA wires

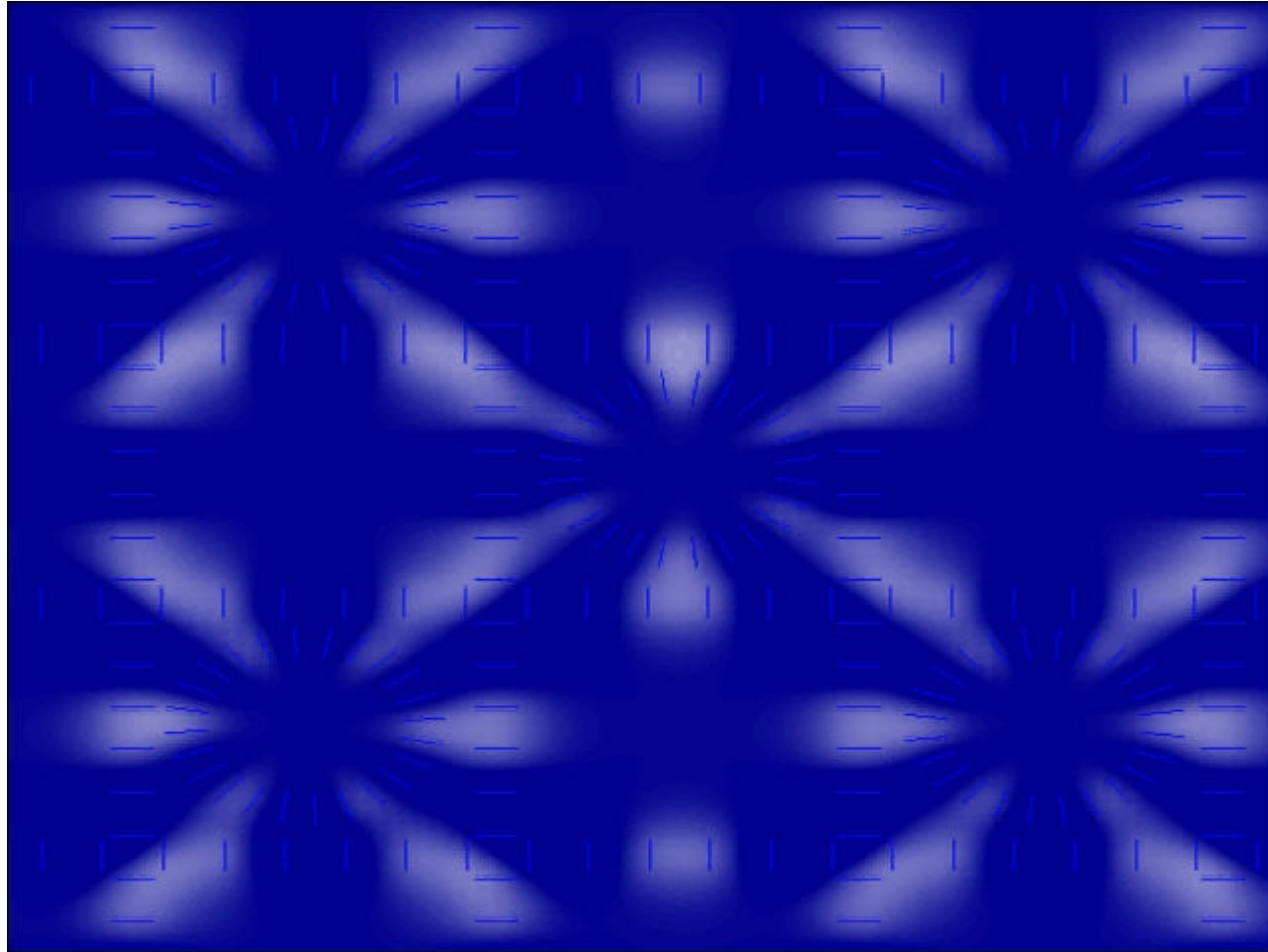
Internal redundancy yields defect tolerance.



# Molecular circuits and clocking wires



# Universal floorplan



# Outline

- Introduction
- QCA paradigm
- Implementations
  - Metal-dot QCA
  - Molecular QCA
- Energy flow in QCA
  - Power gain
  - Power dissipation and erasure
  - Bennett clocking



# Power Gain in QCA Cells

- Power gain is crucial for practical devices because some energy is always lost between stages.
- Lost energy must be replaced.
  - Conventional devices – current from power supply
  - QCA devices – from the clock
- Unity power gain means replacing exactly as much energy as is lost to environment.

**Power gain  $> 3$  has been measured in metal-dot QCA.**





# Minimum energy for computation

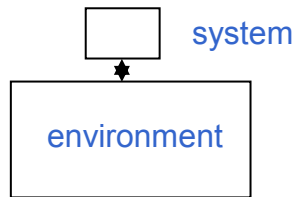
- Maxwell's demon (1875) – by first measuring states, could perform reversible processes to lower entropy
- Szilard (1929), Brillouin (1962): *measurement* causes  $k_B T \log(2)$  dissipation per bit.
- Landauer (1961, 1970): only *erasure* of information must cause dissipation of  $k_B T \log(2)$  per bit.
- Bennett (1982): full computation can be done without erasure.

logical reversibility  $\Leftrightarrow$  physical reversibility



# Theoretical description

## Coherence vector formalism



Extract the real degrees of freedom from the density matrix

real vector  $\vec{\lambda}$

$$\lambda_i = \text{Tr}(\hat{\rho} \hat{\lambda}_i)$$

$\hat{\lambda}_i$  are the  $n^2 - 1$  generators of SU(n), n=2,3

Equation of motion

$$\frac{d\vec{\lambda}}{dt} = \mathbf{\Omega} \vec{\lambda} + \frac{\vec{\lambda} - \vec{\lambda}_{ss}}{\tau}$$

$$\mathbf{\Omega}_{ik} = \sum_j f_{ijk} \Gamma_j$$

$$\Gamma_j = \left(\frac{1}{\hbar}\right) \text{tr}(H \hat{\lambda}_j)$$

$f_{ijk}$  : structure constants of SU(n)

$$\vec{\lambda}_{ss} = \text{tr}(\rho^{eq} \hat{\lambda}_i)$$

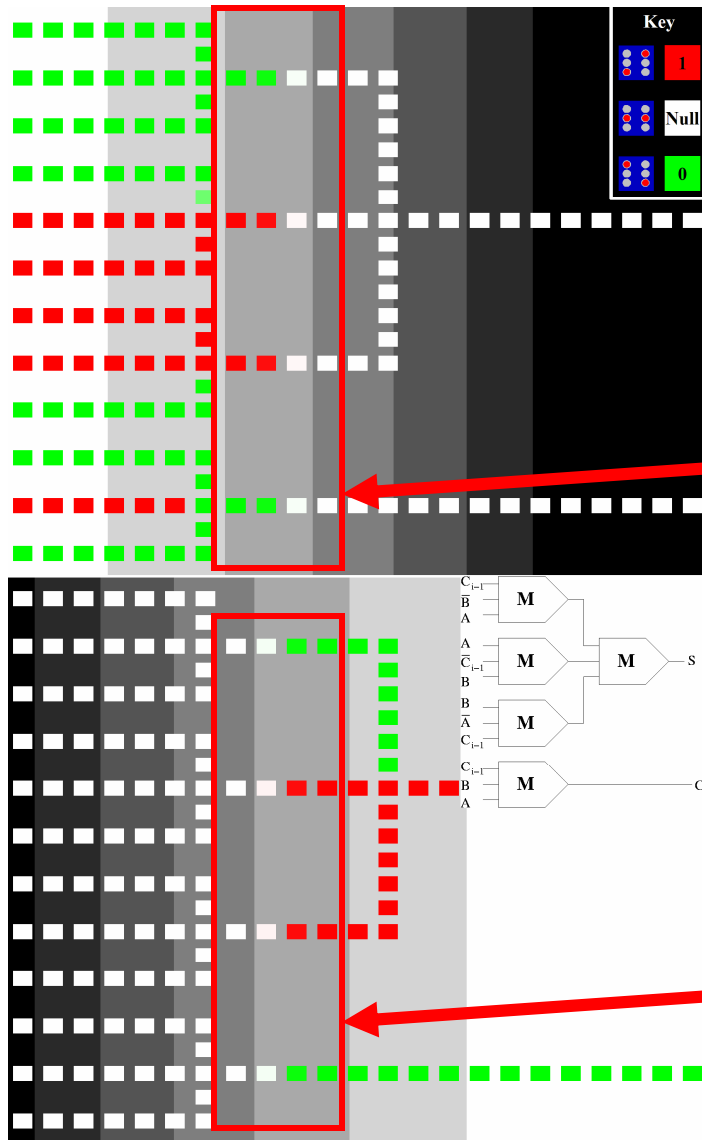
$$\rho^{eq} = \frac{e^{-\beta H}}{\text{tr}(e^{-\beta H})}$$



# Computational wave: adder back-end



# The computational wave



Computation happens here.

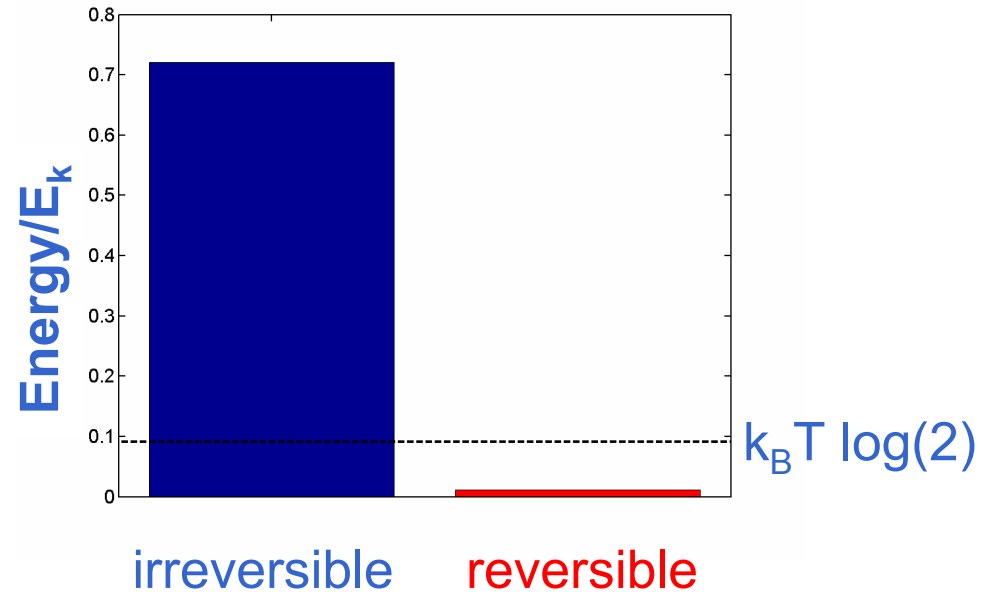
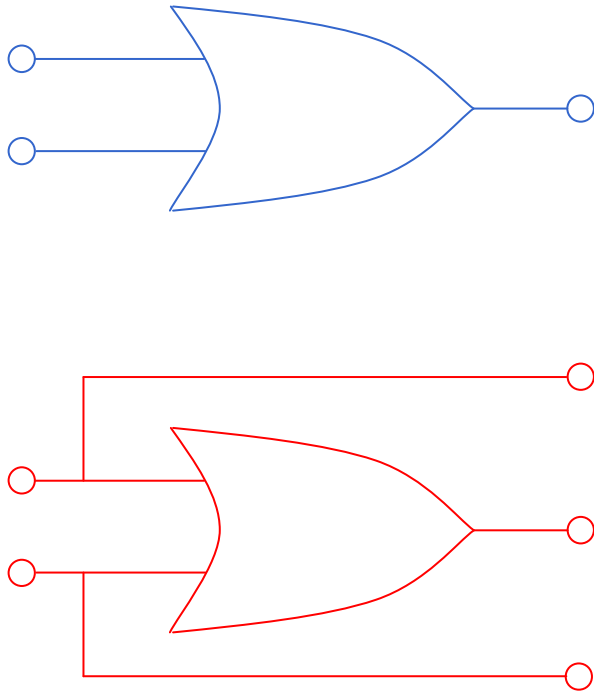
Dissipation (if any) happens here.



# Landauer clocking of QCA



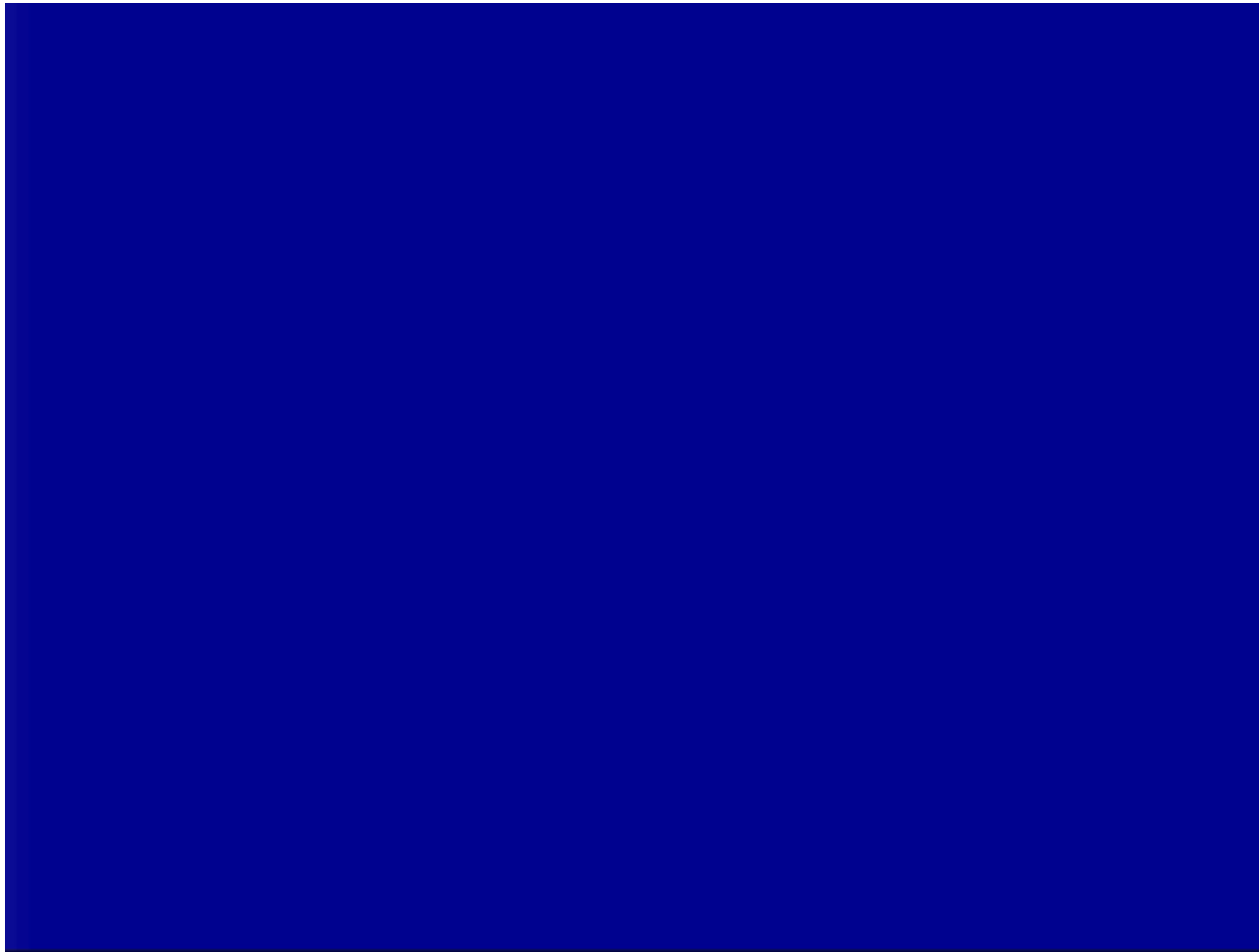
# Bennett-style circuit reversibility



Direct time-dependent calculations shows: Logically reversible circuit can dissipate much less than  $k_B T \log(2)$ .



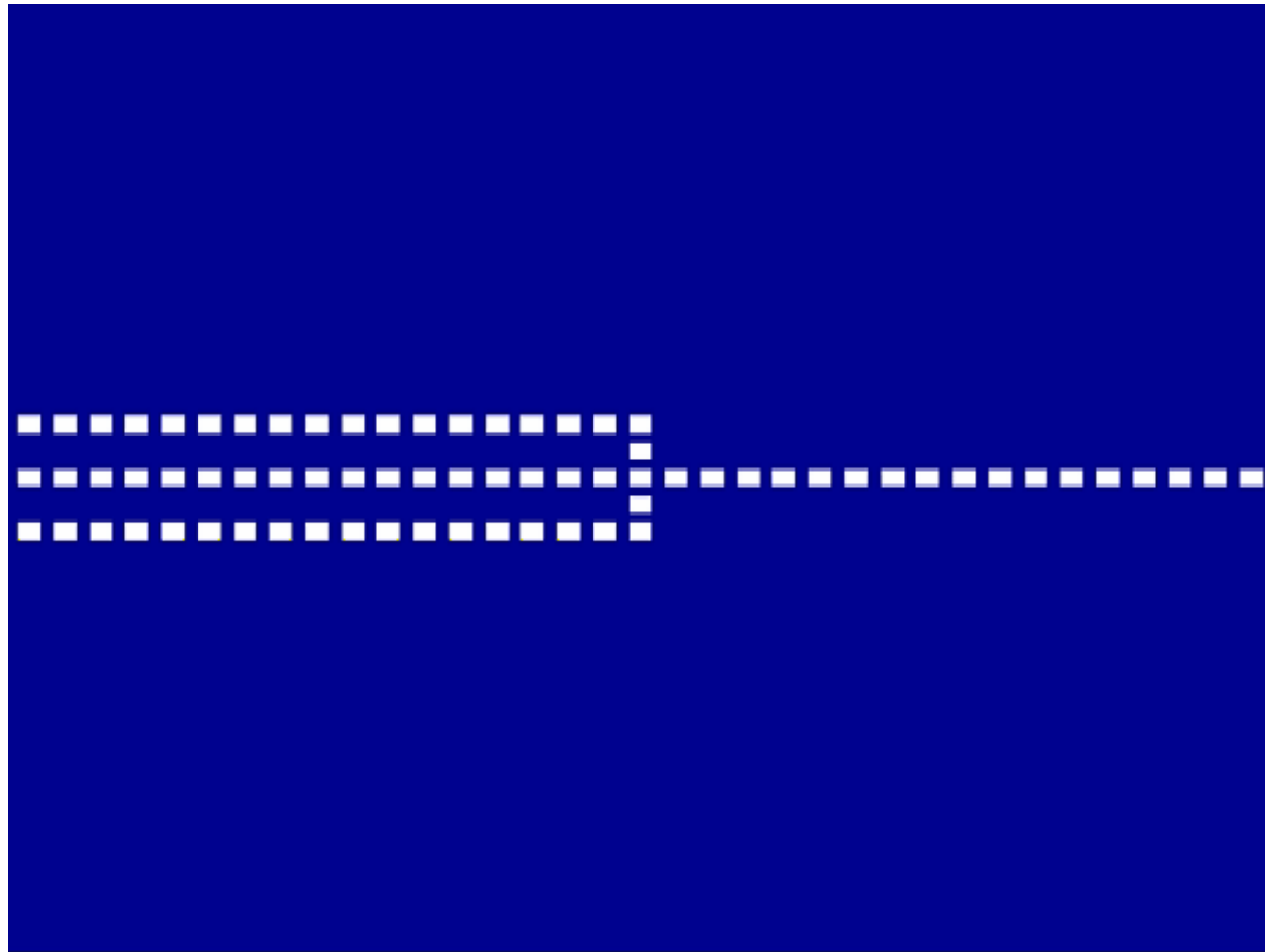
# “Bennett clocking” of QCA



Output is used to erase intermediate results.



# Bennett clocking of QCA

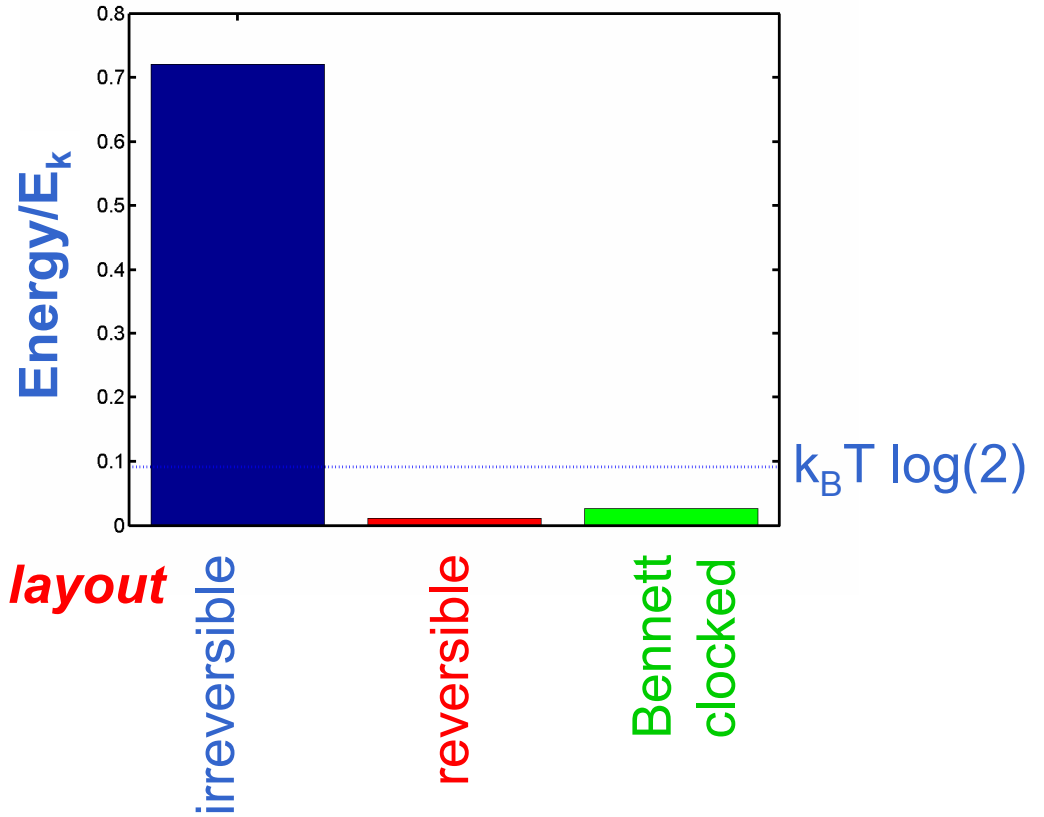
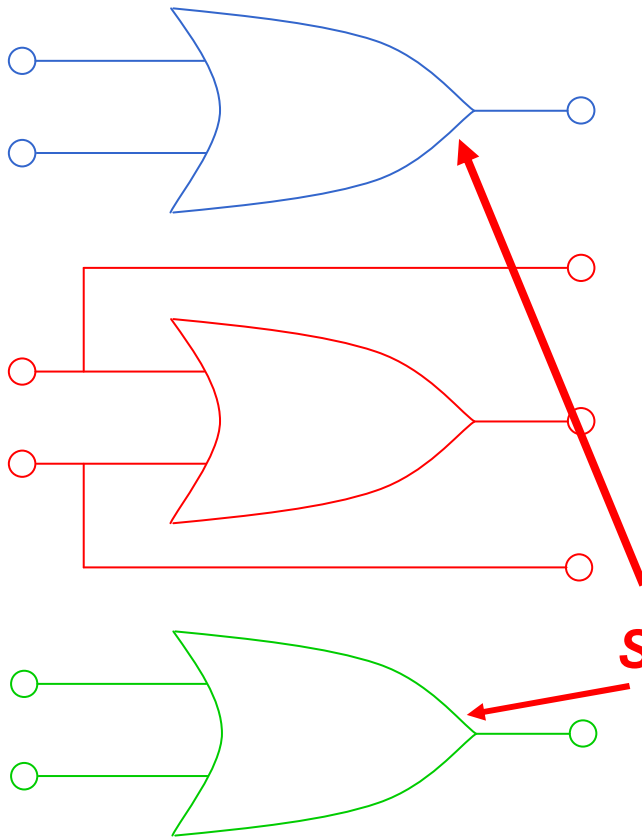


For QCA no change in layout is required.





# QCA gate: reversible/irreversible



Direct time-dependent calculations shows: Bennett-clocked circuit can dissipate much less than  $k_B T \log(2)$ .



# Power dissipation limits

- QCA can operate at the theoretical limits of low power dissipation.
- For regular clocking: must dissipate  $k_B T \log(2)$  for each erased bit.
- For Bennett-clocking: no fundamental lower limit. Cost: half clock speed, more complicated clocking.
- Makes extreme high densities possible—clocking type is in design space.



# Doesn't adiabatic mean slow?

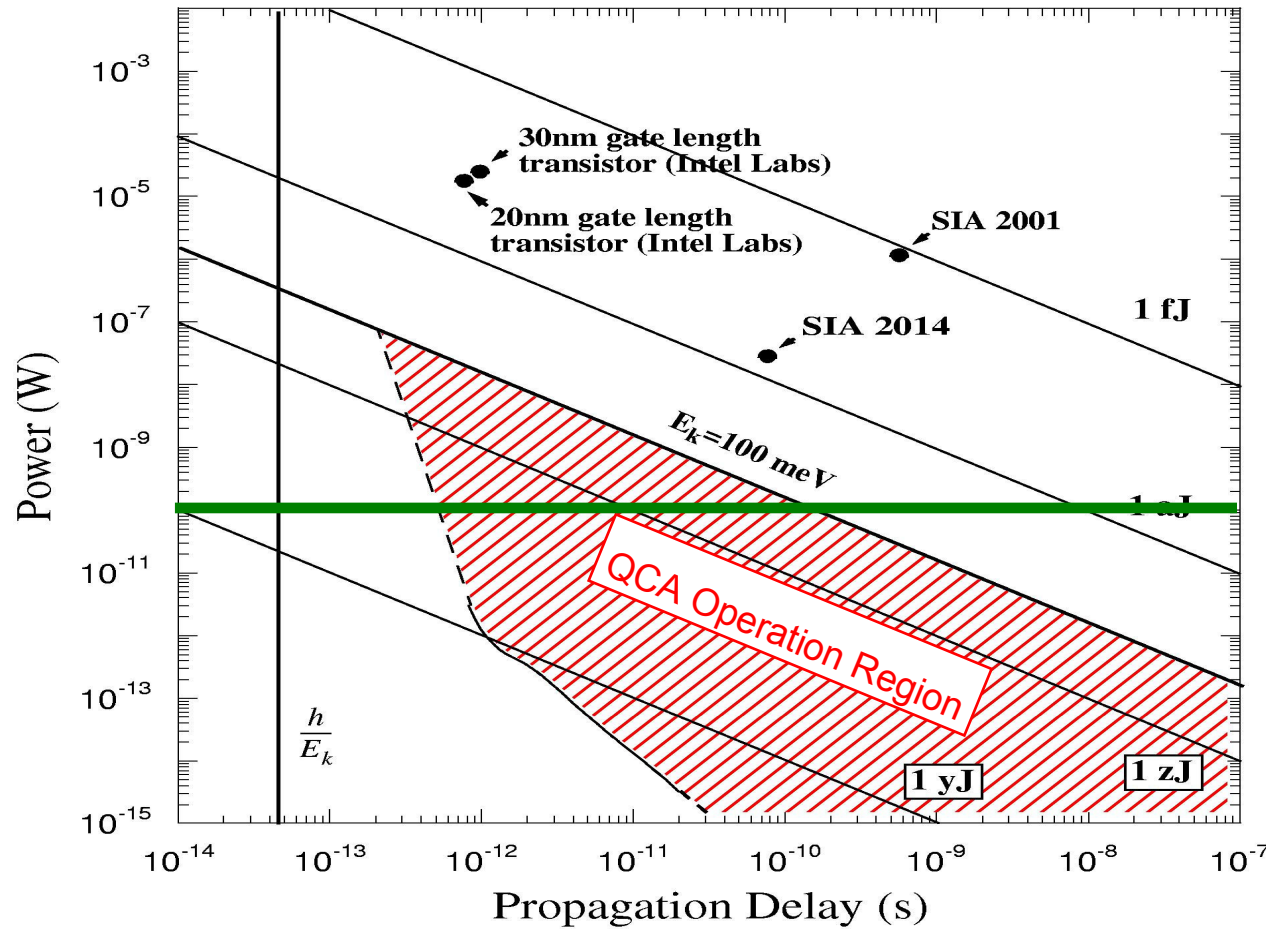
Slow compared to what?

- For conventional circuits,  $\omega < RC$
- For molecular QCA, slow compared to electron switching from one side of a molecule to the other

$$f < f_B \sim 10^{15} \text{ Hz} \rightarrow \text{THz operation is feasible}$$



# QCA Power Dissipation



100 W/cm<sup>2</sup>  
@10<sup>12</sup> devices/cm<sup>2</sup>

QCA architectures could operate at densities 10<sup>12</sup> devices/cm<sup>2</sup> and 100GHz without melting the chip.



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# Is Zettaflops computing possible?

Minimum device size: 1 nm x 1 nm

→  $10^{14}$  devices/cm<sup>2</sup>

Maximum switching speed:  $10^{15}$  Hz

Total chip area: 10 cm x 10 cm

Maximum devices that could be switching

=  $10^{14} \times 10^{15} \times 10^2 = 10^{31}$  switches/sec



# Is Zettaflops computing possible?

Downgrade density

$$10^{14} \rightarrow 10^{12} \text{ devices/cm}^2$$

Downgrade speed

$$10^{15} \text{ Hz} \rightarrow 10^{12} \text{ Hz}$$

Total chip area: 10 cm x 10 cm

Gate op/flop  $10^5$

$$\rightarrow 10^{12} \times 10^{12} \times 10^2 \times 10^{-5} = 10^{21} \text{ FLOPS}$$

Possible.... but challenging



# Main Points

- Quantum-dot Cellular Automata (QCA) is transistor-less approach for solving the challenges of
  - Scaling devices to molecular dimensions
  - Avoiding huge power dissipation issues
  - Power gain (lacking in crossbars)
  - Robustness against disorder
- QCA is an example of operating at the ultimate limits of low power dissipation.
- Direct calculation of the time evolution of QCA arrays illustrates the Landauer Principle. (no hand-waving required)
- QCA can be operated in a Bennett-clocking mode.
- Zettaflops operation is *conceivable*

