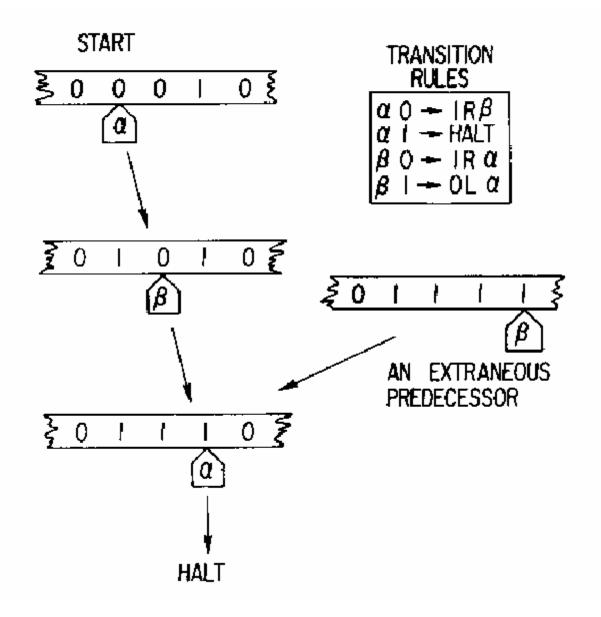
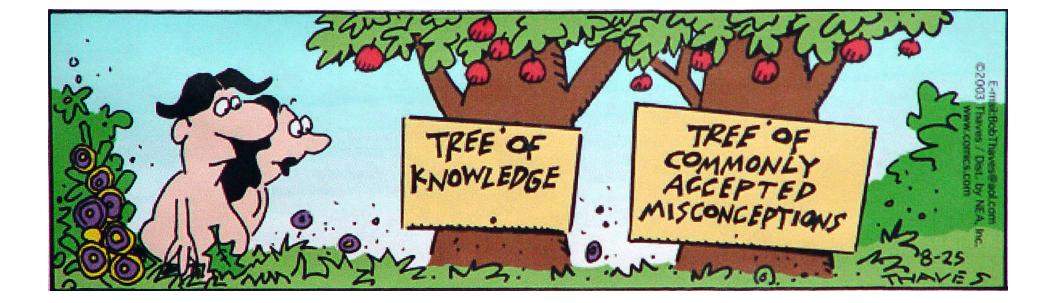
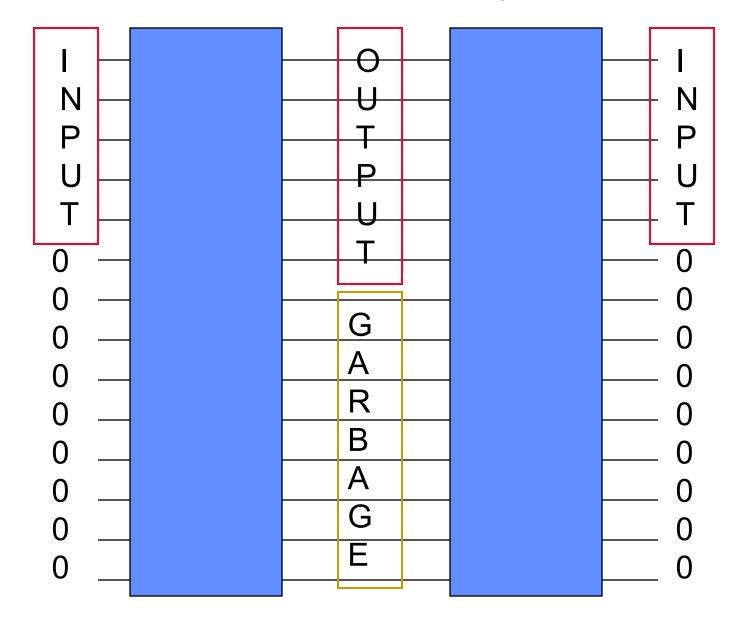
## Turing machine, illustrating logical irreversibility

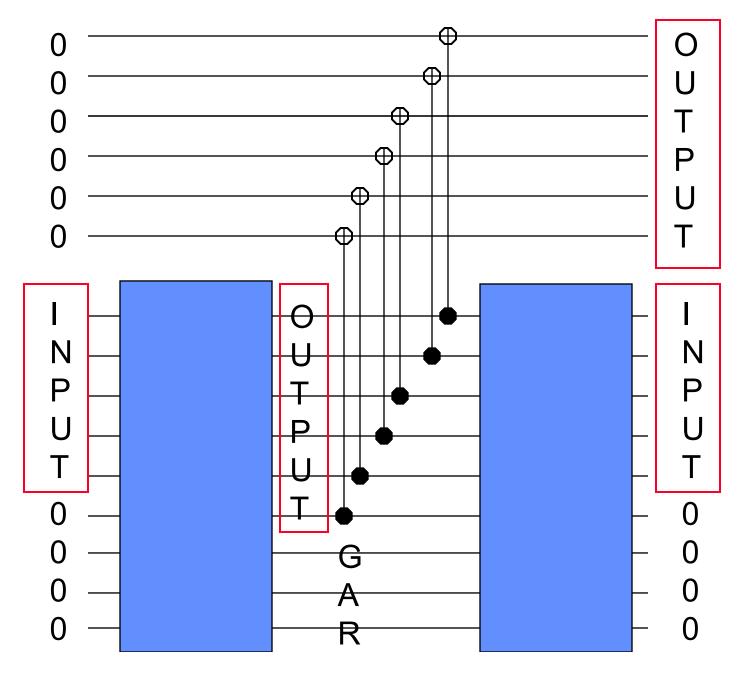




Time-Efficient Space-Inefficient reversible simulation of an irreversible computation



## Using CNOTs to copy output before undoing computation



Another view of the Time-efficient, space-inefficient simulation



T steps of irreversible computation are simulated in 2T steps of reversible computation, using O(T) extra memory for temporary storage of intermediate results.

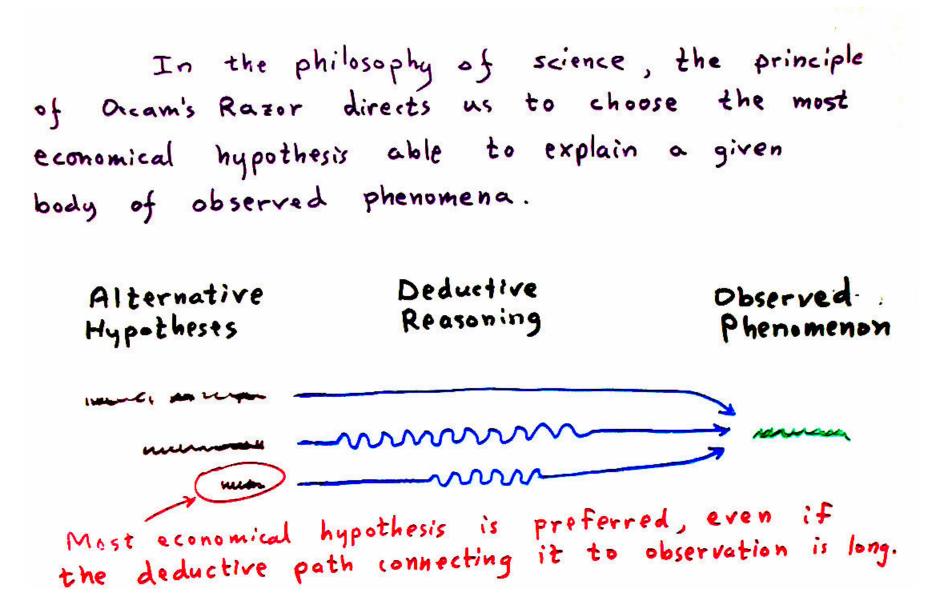
Like laying down a row of stepping stones to cross a river, then removing them. A stepping stone may be placed or removed only when its predecessor is present.

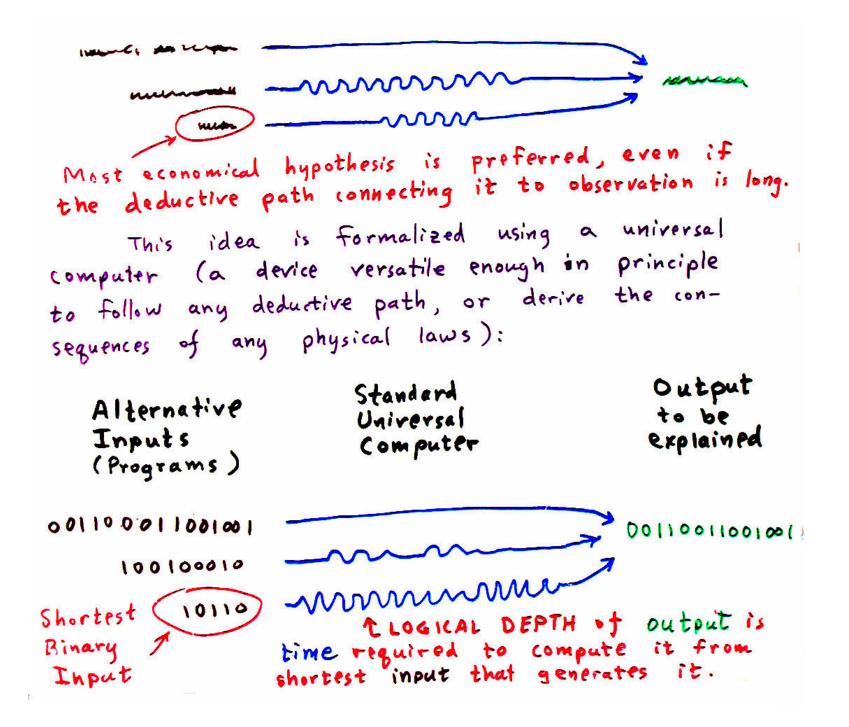
Trading time for space: By doing and undoing steps in a hierarchical manner,  $T=2^m$  steps of irreversible computation can be simulated in  $3^m$  reversible steps using O(m) temporary intermediate storage.



More generally, this type of argument shows that for all  $\varepsilon > 0$ , an irreversible computation using time T and space S can be reversibly simulated in time  $\propto T^{1+\varepsilon}$  and space  $\propto S \log T$ . A still more space-efficient simulation runs in exponential time and linear space.

Occam's Razor & the notion of Logical Depth





What did "Information is Physical" mean to Landauer?

1. We ought to think more about physical principles like the 2<sup>nd</sup> law when we are developing a theory of information processing.

2. It is a waste of time for mathematicians to think about things like the  $2^{1,000,000th}$  digit of pi, which have no chance of being calculated in the physical universe.

But what about the  $2^{1,000,000th}$  digit of 1/7?